DEPARTMENT OF OCEAN ENGINEERING

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AN IMPROVED METHOD OF RISK ANALYSIS FOR THE NAVAL SHIP DESIGN PROCESS

by

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ABSTRACT

A new method for quantifying risk in the naval ship design process by means of probability distributions is presented. Risk is equated with uncertainty about the characteristics of the design. The causes of uncertainty in a ship synthesis model are discussed and analytical methods for combining probability distributions based on work in the cost analysis field are presented. Additionally, recommendations are made on how to implement this methodology in the Navy's ASSET ship synthesis model and an example is given of how the results from a tradeoff study using these methods would be presented to a decision maker. The methodology will give a clearer picture of the type and sources of risk to a decision maker and has applications in several phases of ship design. Finally, recommendations for further research and implementation are given.

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CHAPTER 1

INTRODUCTION

The modern naval ship is an extremely complex system, in fact it has been suggested that a modern aircraft carrier is the most complex engineered system on earth. Due to this complexity the design and acquisition process is by necessity an iterative one as shown in Figure 1.

Another characteristic of this design process is that there is normally more than one possible solution to a problem. Trade-off studies are conducted to explore these alternative solutions and provide the facts for the decision makers who must choose among these alternatives.

There are several factors that influence the decisions and are evaluated in the tradeoff studies. Some of these are: cost (both acquisition and operating), impact on the ship's characteristics (size, installed capacities, etc.) and performance (e.g. speed, endurance, detection range, etc.), and risk (cost, schedule or technical).

The purpose of this thesis is to explore a new method for classifying and measuring risk with emphasis on the technology assessment and feasibility phases of the ship design process. Figure 2 shows a time line for these different phases. This new method is based largely on work done by cost analysts in the aerospace industry [1,2,3].

Figure 1. Iterative Decision Process

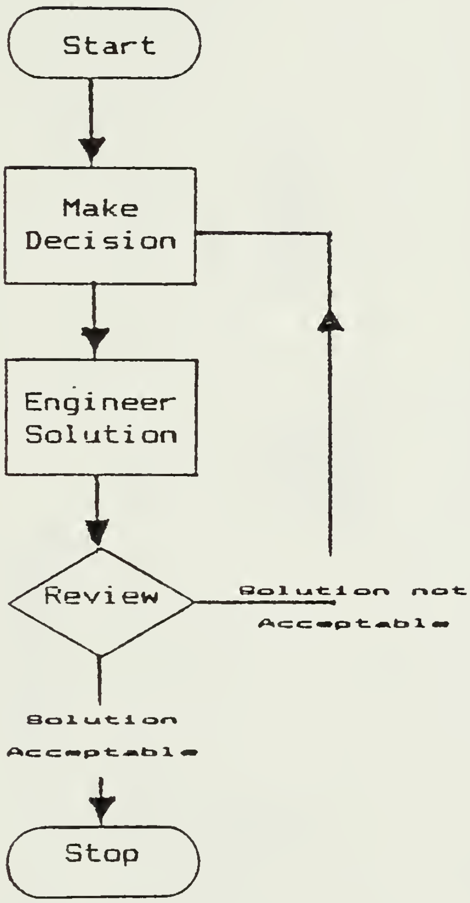
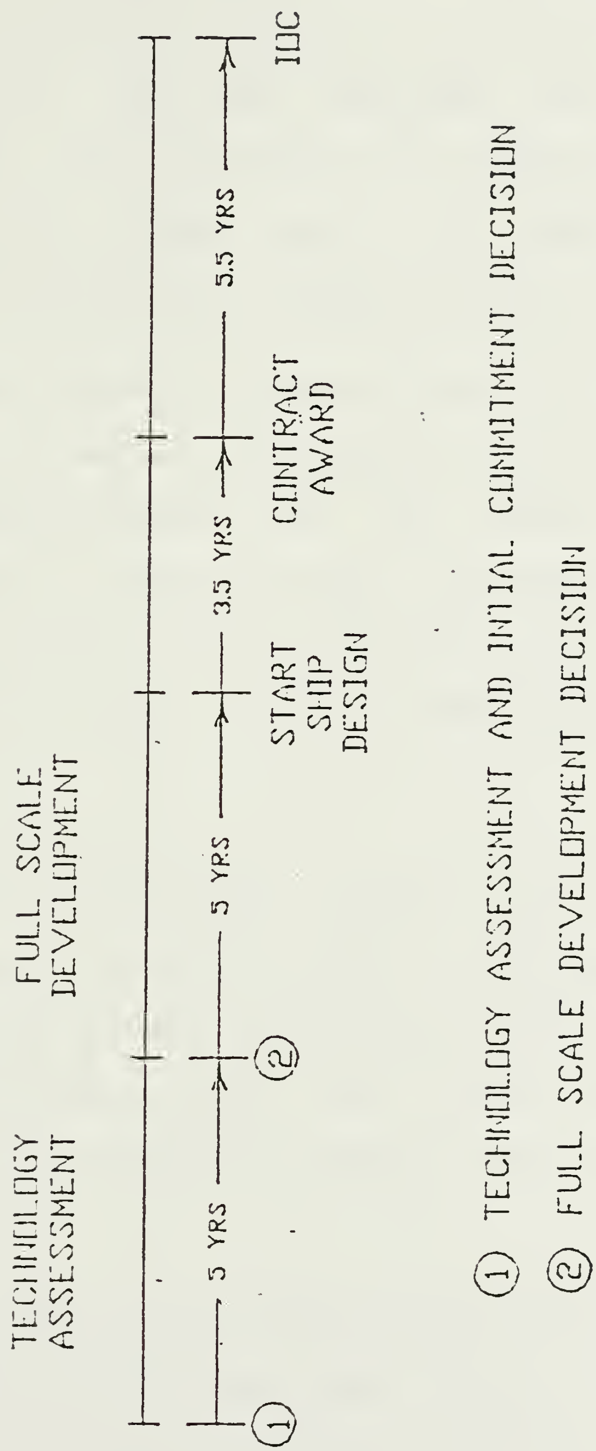


Figure 2. Ship Design Timeframe



Risk is defined in the dictionary as: "a factor, element or course involving uncertain danger or hazard." In the ship design process there is always uncertainty due to factors such as design practices that are based on empiricism because of imperfect knowledge and estimating relationships that are simplified to reduce the effort required for early stage work. In addition, the introduction of new technology brings uncertainty as well.

The current method of handling risk is to start with single point values of the input variables and analyse the design in a deterministic way. The results are then presented as point values and the associated risk is classified on what Dr. Gerald McNichols [4] calls an ordinal or relative scale, usually something like high, medium or low.

This current methodology has several drawbacks.

First, risk is treated as another, separate decision factor instead of being an attribute of the other factors. As will be shown later, the current descriptions of risk are vague and ambiguous compared to the proposed method.

Secondly, the current method requires one value of each input variable to be chosen when there are in fact several or even an infinite number of possible values and combinations of values. This means that only one of the many possible outcomes is treated and the assessed risk is

based on those assumed values. For example, if in a tradeoff study the input values for a new technology are very optimistic, the alternative might be rejected due to high risk whereas the assumption of more conservative values would lead to a lower risk and might still offer an improvement over the baseline technology.

Finally, the current approach has a problem when trying to assess the risk for combinations of innovative items. The question here is something like, "If two items are evaluated as being medium risk when considered individually, is their combination in the same design still medium risk or is it then high risk?"

This thesis proposes to describe risk by means of probability distributions for the various attributes that are considered in the tradeoff studies. Using Dr. McNichol's terminology, this is a cardinal, or quantitative measure of risk. This approach considers that the possible values for input to or output from the design process are continuous across some range.

The use of probability distributions allow several descriptive terms to be used to classify the risk of alternative approaches. Some of these are; mean value, most likely value (mode), lowest and highest possible values, standard deviation, and probability of achieving a certain value. The probability distributions can also be shown graphically, giving the decision maker a more intuitive

feel of these factors. This approach also lends itself to evaluating the effect of combinations of items and the effect of variations in estimating relationships.

Finally, having probability distributions availability would allow the use of Decision Analysis methods using utility functions [5,6,7] which would help decision makers be more consistent.

Figures 3 and 4 give examples of a summary presentation of a tradeoff study using the current and proposed methodologies respectively.

In order to implement this improved method of risk assessment, it is necessary to develop a system for generating these probability distributions. The rest of this thesis is devoted to that task.

Figure 3. Current Tradeoff Presentation

	BASELINE TECHNOLOGY	NEW TECHNOLOGY
Full Load Displacement	BL	- 109 LTONS
Fuel for 4500 NM	BL	- 43 LTONS
Sustained Speed w/ 42,000 SHP	BL	+ 0.5 KTS
Acquisition Cost	BL	- $5M
Risk	LOW	MEDIUM

Figure 4. Proposed Tradeoff Presentation

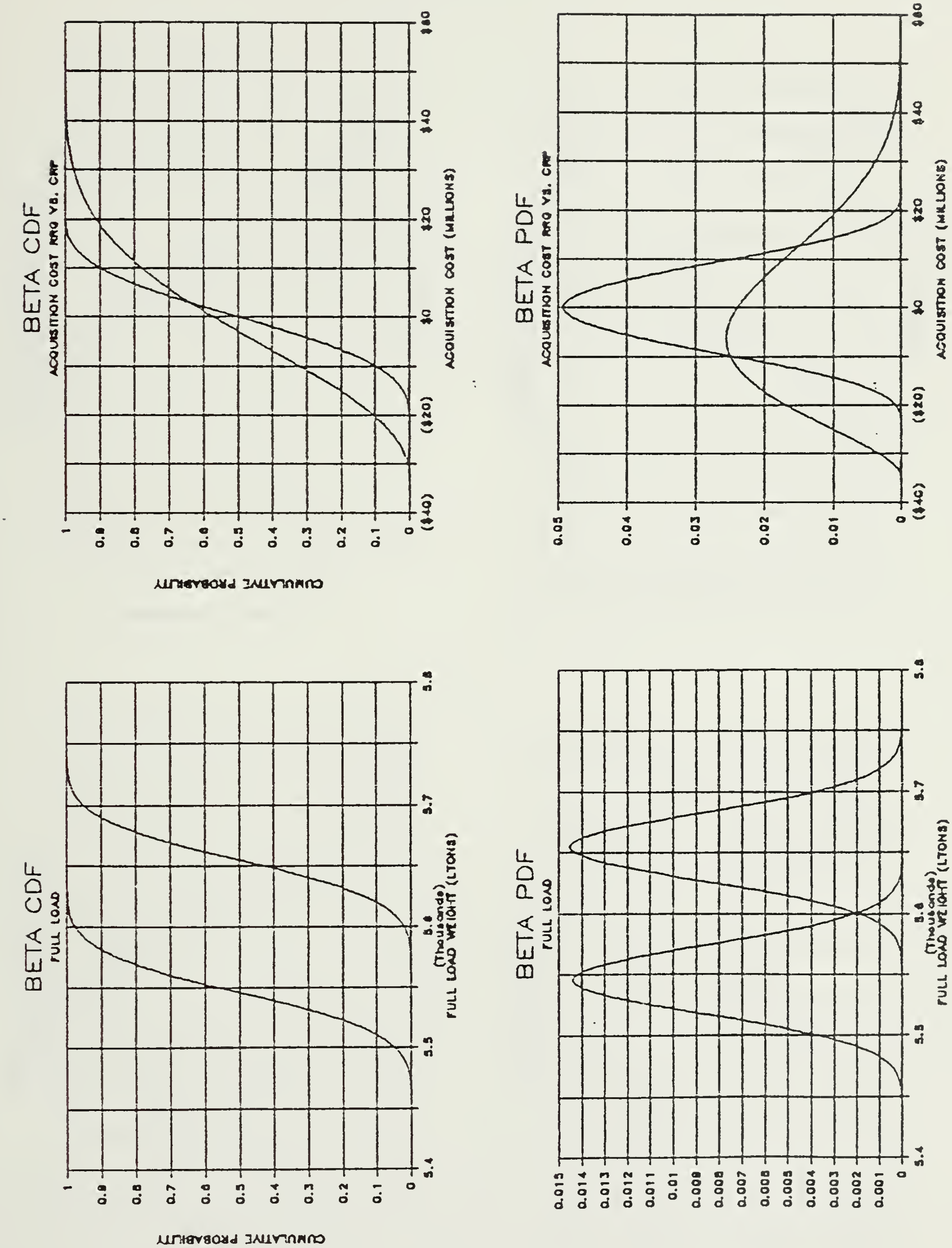
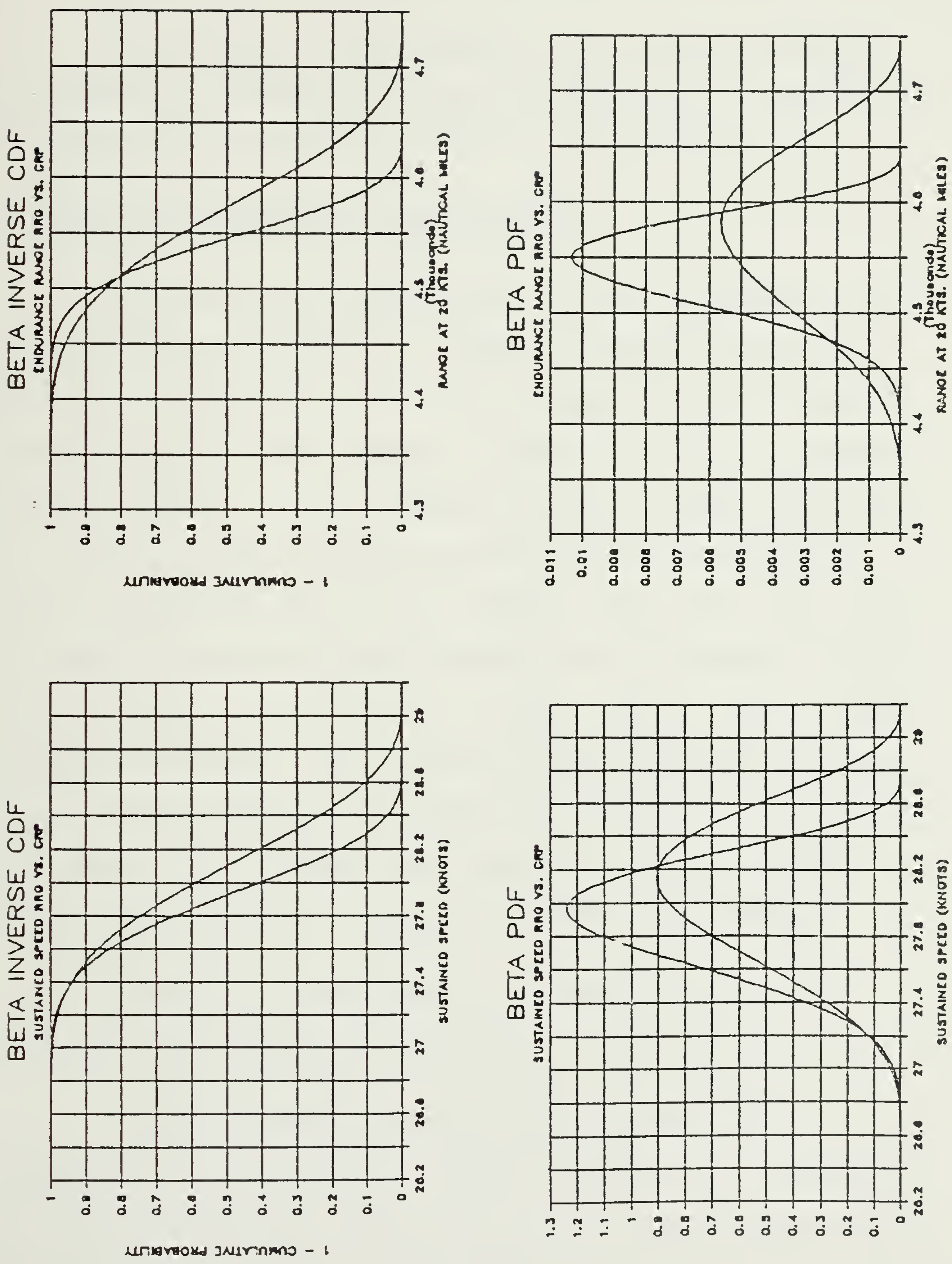


Figure 4. (continued)



CHAPTER 2

BASIC THEORY OF PROBABILITY DISTRIBUTIONS

The purpose of this chapter is to provide a review of the basics of probability distributions and a discussion of some special families of probability distributions.

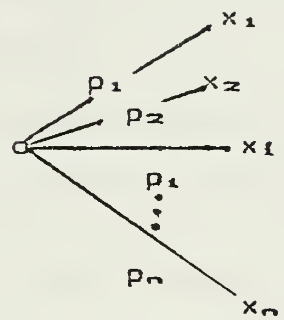
2.1 Discrete Probability Distributions

A probability distribution is used to describe the chance or probability of a random variable taking on a particular value. The form of the probability distribution most familier to the average person is for a *discrete* random variable. In the simplest form this takes the form of flipping a coin to determine heads or tails. Normally it would be expected that on the average, the coin will come up heads 50 percent of the time and tails 50 percent of the time although for an unfair or loaded coin, these values could be different. This example can be extended to a case with a larger number of outcomes such as the roll of a pair of dice or the chances that each of the 26 teams in professional baseball has of winning the World Series.

Figure 5 gives a generic representation of a discrete probability distribution.

The most important point here is that these probability distributions are defined for mutually exclusive outcomes and that the sum of the probabilities for all mutually exclusive outcomes is 1.

Figure 5. Discrete Probability Distribution



$$\sum^{n} p_i = 1$$

2.2 Continuous Probability Distributions

This thesis will be dealing with *continuous* random variables instead of discrete ones. For a continuous variable, the probability of a particular value occurring is infintesimally small. For this reason the probability must be expressed in terms of a range. Standard practice is to express the probability as a *cumulative distribution function* (cdf), P(x), which gives the probability that the random variable is less than or equal to x.

The cumulative distribution function has the following properties:

(1) It is monotonically increasing with $P(-\infty) = 0$, $P(+\infty) = 1$.

(2) The probability that the random variable lies within the range x_1 to $x_2 = P(x_1) - P(x_2)$.

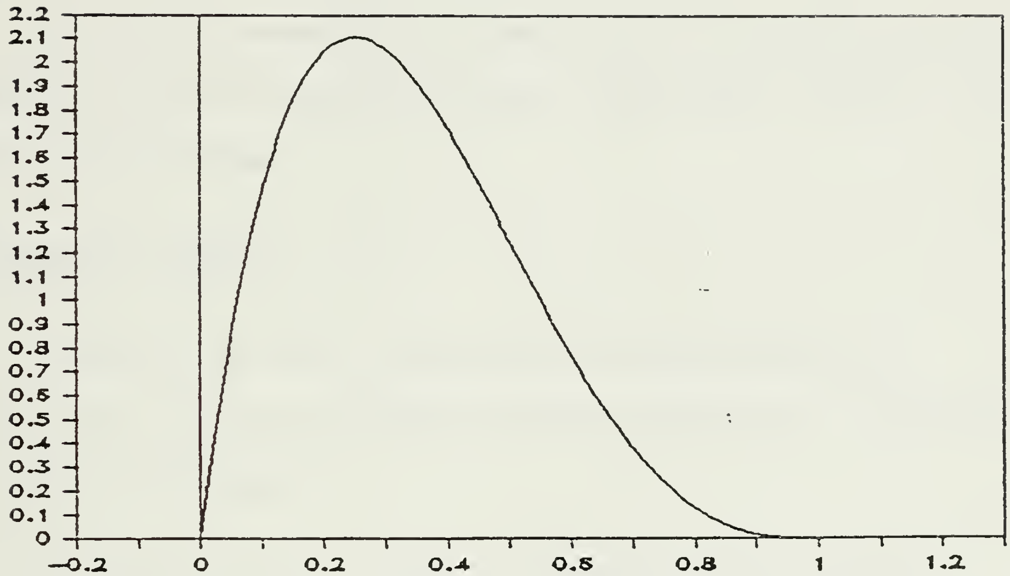
Analogous to the discrete probability distribution discussed before is the *probability density function* (pdf), p(x). The pdf is the derivative of the cumulative distribution function. This means that the total area under the pdf is equal to 1 just as the sum of the discrete probabilities is equal to 1.

A graph of a pdf (Figure 6) conveys such information as:

(1) upper and lower bounds on possible values of the variable.

(2) most likely value (mode).

Figure 6. Probability Density Function



$$\int_{-\infty}^{+\infty} p(x)\, dx = 1, \quad P(x_1) = \int_{-\infty}^{x_1} p(x)\, dx$$

(3) spread or variance.

(4) skewness or non-symmetry.

Further information on these basics can be found in any probability and statistics text. References [8] and [9] are cited as examples.

2.3 Parametric pdf's

Families of pdf's exist that can be described parametrically. Some of these families and their properties will now be discussed.

The best known of these parametic pdf's is the *Gaussian* or normal distribution (Figure 7). This distribution is applicable to many situations occuring in the real world. It is described by the equation:

$$p(x) = 1/\sigma\sqrt{2\pi}\, \exp[-(x-\mu)^2/2\sigma^2]$$

This distribution has two parameters, the mean, (first moment), and the variance, σ^2 (second central moment).

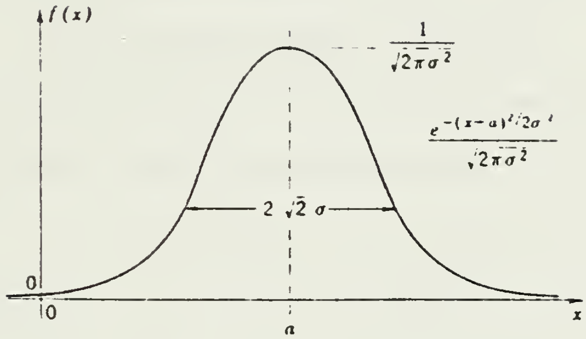
Some other properties of the Gaussian distribution are:

(1) it is symmetric (has zero skew).

(2) its limits are plus and minus infinity.

(3) 99.74% of the area is within the range between plus and minus infinty.

Figure 7. Gaussian PDF



Another pdf which has been extensively used by cost analysts [10] is the *Generalized Beta* distribution defined by the function:

$$p(x;\alpha,\beta,a,b) = [\Gamma(\alpha+\beta+2)/(\alpha+1)(\beta+1)b][(x-a)/b][1-(x-a)/b]$$

a,b,α,β are real numbers, $b \geq 0$, $\alpha,\beta > -1$

$\Gamma(u)$ is the gamma function

This distribution has four parameters which are a, b, , and . The Beta pdf is defined over a finite range from a to a + b. The shape of the distribution depends on the values of the exponents alpha and beta as shown in Figure 8. The shape that is of the most interest to this thesis is the unimodal one which corresponds to α and $\beta > 0$.

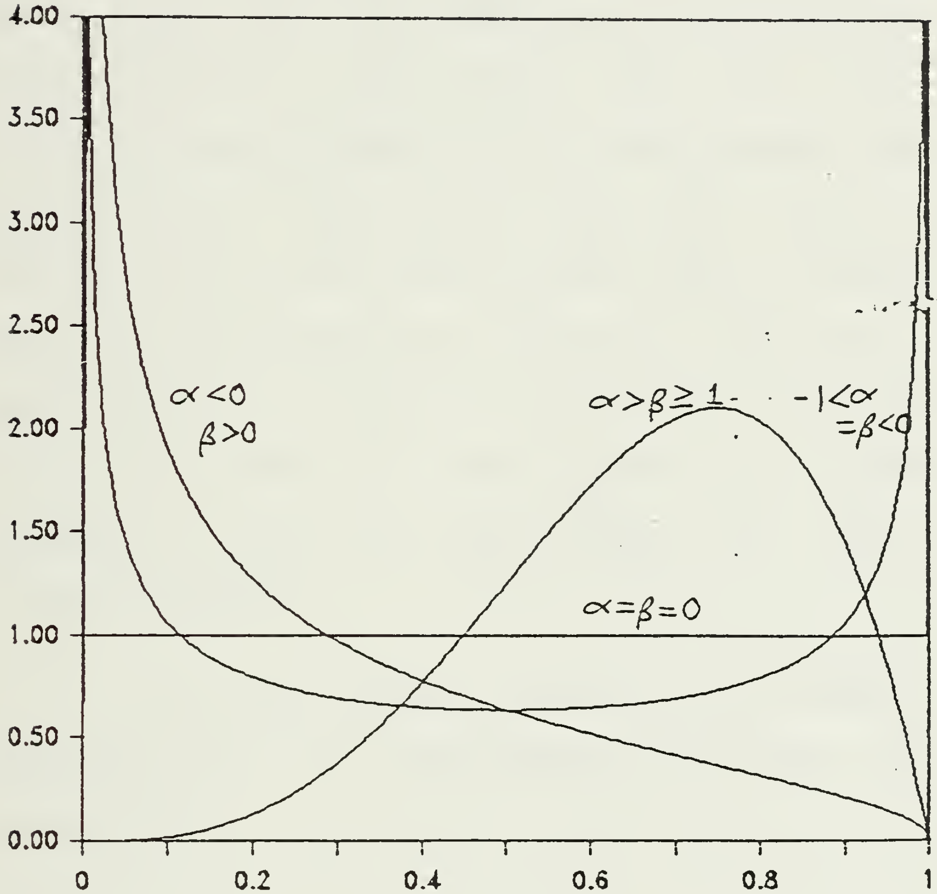
Another distribution family used by cost analysts [11] is the *Generalized Gamma* distribution which is also a four parameter pdf defined by:

$$p(x;\beta,\gamma,\ ,k) = \gamma/\Gamma(\alpha)\ \beta^{-\alpha\gamma}\ (x-k)^{\alpha\gamma-1}\ \exp[-((x-k)/\beta)^{\gamma}]$$

α,β,γ,k positive real numbers, $x > k$

This distribution has a finite lower limit k and an upper limit of $+\infty$. It is also skewed to the right. Several commonly used distributions can be expressed as special cases of the Generalized Gamma including the Exponential, Weibull, Chi-Squared and Rayleigh distributions.

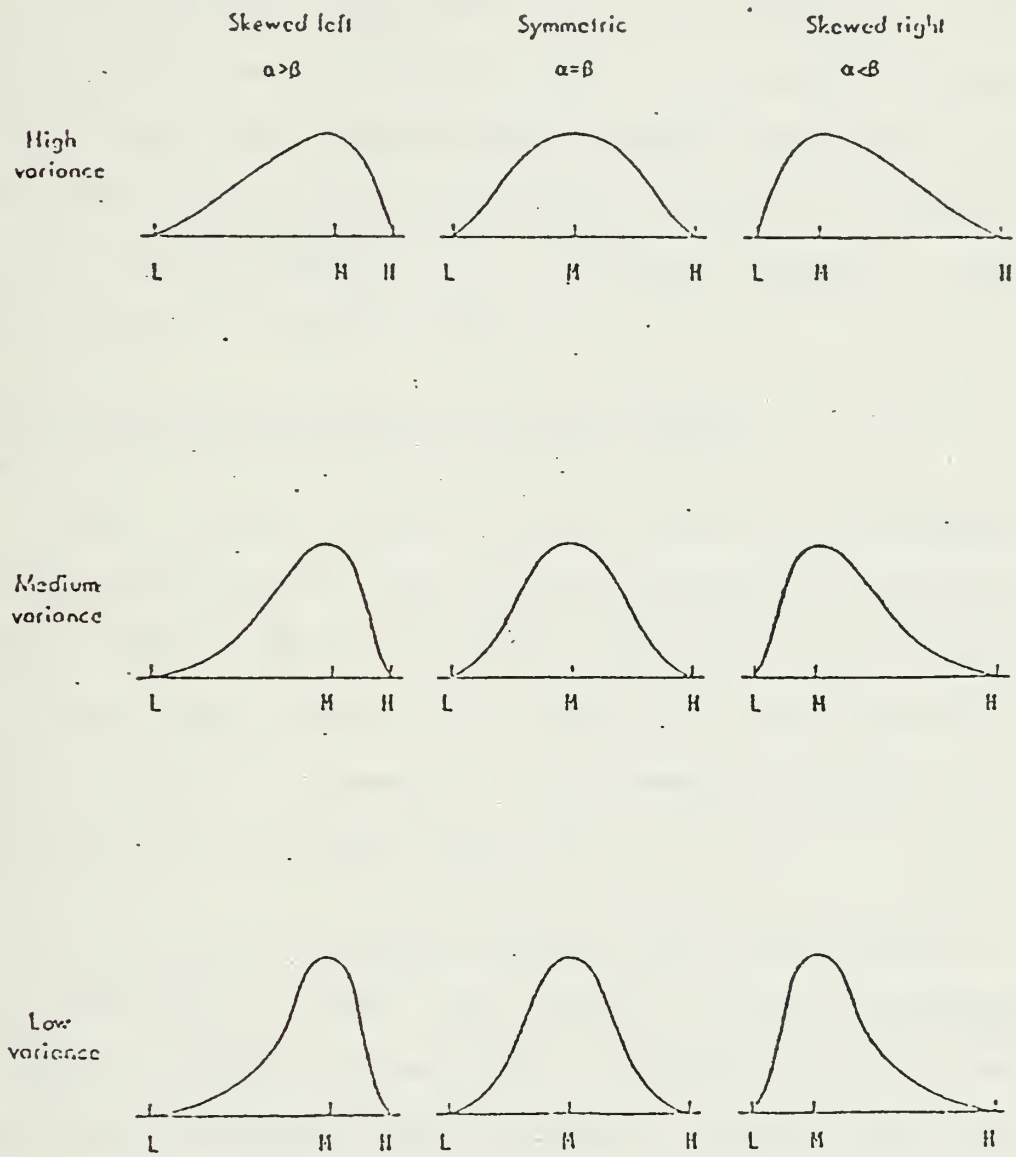
Figure 8. Variation of Beta Distribution Shape with Parameters



One of the goals of this thesis is to express the characteristics estimated in a tradeoff study in the form of probability distributions. In addition it will be necessary to handle some input and intermediate data as probability distributions. The Beta distribution was chosen for this purpose for several reasons. First, it has finite upper and lower limits which makes more sense for the type of characteristics that will be dealt with. Secondly, of the three distributions discussed, the Beta is the most flexible in terms of the shapes it can assume as shown in Figure 9. The Gaussian is limited to symmetric shapes and the Gamma to only right (positive) skew.

The next chapter will discuss how these characteristics are estimated and the causes of their uncertainty.

Figure 9. Shape Variations for Unimodal Beta PDF [1]



CHAPTER 3

UNCERTAINTY AND THE SYNTHESIS MODEL

As previously discussed in the introduction, a ship is an extremely complex system with an iterative design process and more than one feasible solution for a given set of requirements. For early stage design work to establish the gross characteristics of a feasible design a *Ship Synthesis* model is normally used.

3.1 Basic Principles of Ship Synthesis Models

Table 1 shows the major areas that must be balanced in a ship design. These areas are interrelated so iteration is required both within and between modules. For example, the installed power affects the weight of the propulsion subsystem but the assumed displacement affects the installed power requirement and so on.

This iterative process was done for many years by hand but beginning in the late 1960's it was automated using digital computers. The early synthesis models were straightforward adaptations of the hand methods but later versions have added features that would not be practical for manual calculations. The advent of these computerized models also allow the exploration of many more alternatives than were considered with the previous methods. In a given design project, these studies are used for optimizing the configuration of the design [12].

Table 1. Balance Requirements

AREA	REQUIREMENT
Energy	Installed power is sufficient to achieve required sustained speed at design displacement. Installed electric plant sized for expected load. Fuel allocated sufficient to meet endurance requirement.
Space	Volume and deck area available equal to or greater than volume and deck area required.
Weight	Sum of subsystem and load weights equal to assumed displacement.
Stability	Metacentric height (GM) within acceptable range.

The input into a synthesis model is a set of requirements covering items such as physical characteristics of the payload, manning, sustained speed, endurance speed, and endurance range. The model then uses a set of estimating relationships for the characteristics appropriate to the four areas in Table 1 and a set of logic rules to achieve a balanced design. Finally, the synthesis model outputs selected characteristics of the final, balanced design [13]. Table 2 lists some currently available synthesis models along with their applicability.

Besides being used for actual ship design, synthesis models are useful for evaluating the impact of new technology and design standards. ASSET is particularly intended for this application. Goddard's thesis [17] is a reference for this application.

3.2 Causes of Uncertainty

The values calculated by a synthesis model are uncertain for one or both of two reasons. First, most of the estimating relationships are based on regression analyses, normally with only one or two independent variables. These analyses must use parameters that are available at an early stage of ship design [18] and therefore can only give a rough estimate of the value.

Table 2. Available Synthesis Models

MODEL	SOURCE	APPLICABILITY
REED	MIT [14]	Destroyer Type 1,700 to 17,000 tons
ASSET	DTNSRDC [15]	Monohulls SWATH Hydrofoils
DD08	NAVSEA [16]	Destroyer Type
CV02	NAVSEA	Aircraft Carriers
LL01	NAVSEA	Amphibious Warfare Ships

An example of this type of uncertainty is the ASSET estimating relationship for Firemain Weight, SWBS Weight Group 521:

W521 = 8.0 E-5 (Total Ship Volume)

Secondly, the input values for an estimating relationship may and probably will not be known with certainty. Examples of this is the weight and power consumption of a new weapon system that is part of the payload or a relationship that has as its input a value calculated by a regression relationship.

An example of this is the calculation for the superstructure weight, SWBS group 150:

W150 = (Deckhouse Volume)(Deckhouse Structural Density)

The exact value of the density is unknown and in certain cases the volume may be as well.

These two causes of uncertainty can also occur together in the same estimating relationship. How to treat these cases mathematically will be discussed in the next chapter.

The above discussion shows that even in a design based completely on existing technology, a synthesis model will return numbers that are not certain. This fact has been recognized previously and therefore a *margin* based on historical data is normally added [19, 20]. Part of the

purpose of this thesis is to provide a better tool for deriving this margin, especially when using new technology in the design.

The next chapter will discuss analytic methods of treating these causes of uncertainty and Chapter 5 will discuss the implementation of those methods in the ASSET synthesis model.

CHAPTER 4

METHODS OF MATHEMATICALLY HANDLING UNCERTAINTY

4.1 Uncertainty from Regression Analysis

The main basis of estimating relationships for a synthesis model is by some form of regression analysis of data from previous designs. This is particularly true for the weight and volume estimating relationships.

The goal of regression analysis is to find a functional relationship between a value and the factors that affect it. The form most commonly used the "least squares fit" method which was developed for the experimental sciences.

In the simplest form, the least squares method is based on assuming a functional form of the relationship of the form:

$$y_i = a_1 x_i + a_0 + \varepsilon_i$$

where ε is a random variable giving the error

Minimizing the sum of squares of deviations of the data points from the assumed line the following system of equations is derived and solved for the coefficients a_0 and a_1:

$$a_0 \Sigma x_i + a_1 \Sigma x_i = \Sigma x_i y_i$$

$$a_0 \Sigma x_i + a_1 n = \Sigma y_i$$

where n is the total number of data points

The derivation and solution of these equations is in Appendix A. This approach can be extended to derive relationships involving polynomials, linear combinations of different variables, and through a variable transformation, power, exponential, and logarithmic dependencies. The derivation of these is beyond the scope of this thesis but can be found in Reference [21].

The additive error term has two assumptions [22] made about it in the above analysis:

(1) it has a zero mean about the line.

(2) it has constant variance, independent of x.

In addition, most texts on the subject assume that the error term has a Gaussian distribution. This is not a necessary condition for the derivation to be true, but allows certain statistical tests to be carried out for goodness of fit [23].

When the author first started looking at this area, the assumption of constant variance did not seem logical for the derivation of estimating relationships. For experimental work where conditions are closely controlled the errors are due primarily to measurement errors and can

be expected to be independent of the magnitudes involved. The causes of variations in the estimating process are different and is rooted in the fact that the estimating relationship is a considerably simplified model of the detail design process for a particular area of the ship. Because of this it seems reasonable that for the estimating relationship the magnitude of the possible variations from the regression line would increase as the magnitude of the variables increased.

4.2 Heteroscedasticity

A search through a considerable number of statistics and linear regression texts found no treatment of this assumption of non-constant variance with the exception of Reference [24]. With the aid of Mr. Michael Jeffers of DTNSRDC it was found that this same assumption is commonly used in the field of *Econometrics*, which is concerned with the application of statistical methods to the study of economic data and problems. Reading through several texts on the subject [25,26,27], it was found that econometricians have the same situation as ship designers in trying to model a complex relationship by a simple one using the data most readily available.

This assumption of non-constant variance is called *heteroscedasticity* in the econometric literature and the previous assumption of constant variance is *homoscedasticity*. An example from Reference [28] is the

correlation between consumption expenditures and family income. The example shows that families with an income of \$10,000 have a range of variation of \$4,000 while families with an income of \$50,000 can be expected to have a larger variation.

The least squares procedure previously derived is referred to as Ordinary Least Squares (OLS) and incorporating the assumption of heteroscedasticity results in Weighted Least Squares (WLS).

In WLS a functional relationship is either assumed or known for the variance, i.e. σ^2 proportional to $f(x)^2$. The basic equation is then divided through by the square root of the relationship and the following system of equations are obtained and solved:

$$a_1\Sigma(x_i^2/f^2(x_i)) + a_0\Sigma x_i^2/f^2(x_i) = \Sigma x_i y_i/f^2(x_i)$$

$$a_1\Sigma x_i/f^2(x_i) + a_0\Sigma 1/f^2(x_i) = \Sigma y_i/f^2(x_i)$$

The derivation and solution of these equations is in Appendix A. It can be seen that by assuming $f(x) = 1$ or constant variance, these equations reduce to the OLS case.

An sample regression analysis using this was performed for the relationship discussed in the last chapter for Firemain Weight, SWBS group 521 based on data contained in Reference [29]. The results are presented in Table 3 and Figure 10. For this case $f(x)$ was assumed equal to x which normalizes the variance.

Figure 10. SWBS 521 Firemain Weight

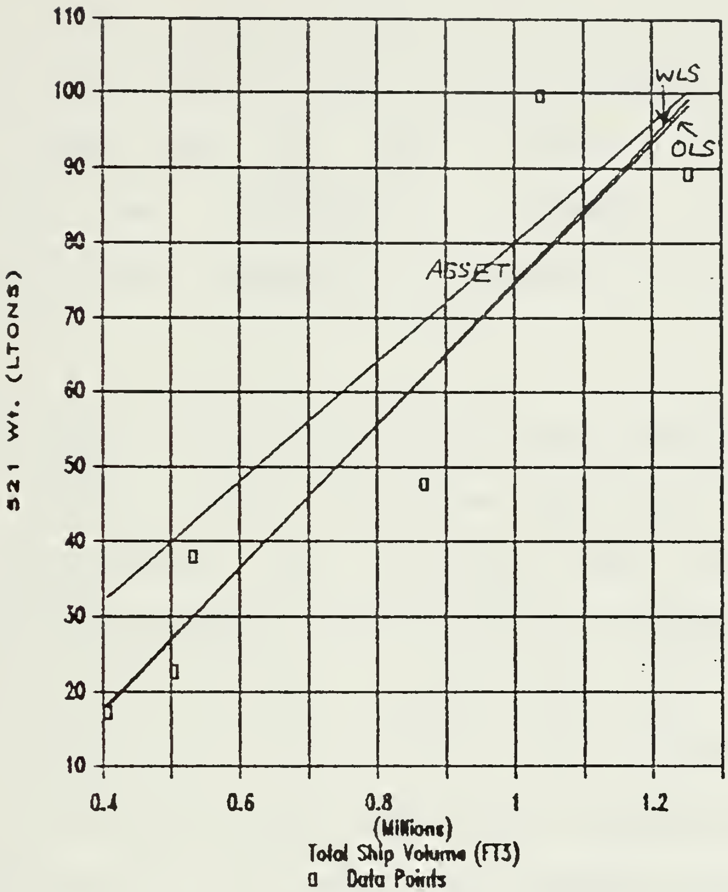


Table 3. Regression Analysis Results for Group 521

ALGORITHM	ERROR
ASSET: 8.0E-5 (TSV)	42.4%
OLS: 9.44 E-5 (TSV) - 19.97	18.5%
WLS: 9.61 E-5 (TSV) - 21.19	18.1%

4.3 Uncertainty from Input Parameters

The other source of uncertainty for the estimating relationship is due to the input parameters. In the synthesis model many of the inputs to estimating relationships are themselves outputs from other relationships and therefore uncertain as previously discussed.

One means of handling this form of uncertainty is by means of a Monte Carlo simulation. Gourley's 1979 thesis [30] on technological risk analysis used this technique with examples of schedule and cost risk. However, a Monte Carlo simulation involves a large number of calculations for each estimating relationship. The procedure is an iterative one and the exact number of calculations is not known beforehand. For the large numbers of estimating relationships used in a synthesis model (approximately 200 in the weight module of ASSET), a method requiring less computational effort is needed.

It can be shown by means of Fourier Transforms [31] that a sum of independent Gaussian random variables defines another Gaussian random variable with mean and variance equal to the sum of the means and variances of the input variables. It has been further proved that this property is true for non-Gaussian random variables as well.

Building on this base, McNichols in his doctoral dissertation [1] devised an analytic method for obtaining

the probability density function for an arbitrary function of random variables. This method utilizes the concept of a rth "additive moment" which is a function of moments of a random variable $C = \Sigma X_i$ with the property that it is equal to the sum of the same function of moments for the X_i's. The first four additive moments in terms of the central moments mu of the X_i's are listed in Table 4.

McNichols then used a Taylor Series expansion of an arbitrary function and derived a formula for what he called a "generalized additive moment". The first four generalized additive moments for a first order approximation are listed in Table 5 and in Table 6 these formulas have been applied to the estimating relationship for Group 150 weight discussed in the last chapter.

In McNichols's dissertation, there are also expansions for a second order Taylor approximation and for dependent variables. The terms due to dependency add to the independent moments. For the purposes of this thesis, dependencies between estimating relationships will not be examined, although it is acknowledged that this would apply in some cases.

The importance of these moments is that they can be used to calculate the parameters of a probability density function. If a Gaussian distribution is assumed only the first two additive moments (mean and variance) are needed since it is a two parameter family. The Beta distribution

Table 4. Additive Moments for $C = \Sigma X_i$

	Additive Moments of C		Expression in Terms of Moments of X_i	
First	$\Lambda_1 \equiv \mu_c \equiv E[C] = \Sigma[\Sigma X_i]$	$= \Sigma E[X_i]$		$= \Sigma\mu_i$
Second	$\Lambda_2 \equiv \mu_c^{(2)} \equiv E[(C-\Lambda_1)^2] = E[(\Sigma X_i-\Sigma\mu_i)^2]$	$= \Sigma E[(X_i-\mu_i)^2]$		$= \Sigma\mu_i^{(2)}$
Third	$\Lambda_3 \equiv \mu_c^{(3)} \equiv E[(C-\Lambda_1)^3] = E[(\Sigma X_i-\Sigma\mu_i)^3]$	$= \Sigma E[(X_i-\mu_i)^3]$		$= \Sigma\mu_i^{(3)}$
Fourth	$\Lambda_4 \equiv \mu_c^{(4)}-3(\mu_c^{(2)})^2 \equiv E[(C-\Lambda_1)^4] - 3\Lambda_2^2 = E[(\Sigma X_i-\Sigma\mu_i)^4] - 3(\Sigma\mu_i^{(2)})^2$	$= \Sigma(E[(X_i-\mu_i)^4] - 3(\mu_i^{(2)})^2)$		$= \Sigma[\mu_i^{(4)} - 3(\mu_i^{(2)})^2$

Table 5. First Order Generalized Additive Moments

Label	GAM	Expression in Terms of Cost Elements
G_{11}	μ_C	$f(\mu_1,\mu_2,\ldots,\mu_n)$
G_{21}	$\mu_C^{(2)}$	$\sum_{i=1}^{n}\left(\frac{\partial f(X)}{\partial X_i}\Big\vert_{\mu}\right)^2 \mu_i^{(2)}$
G_{31}	$\mu_C^{(3)}$	$\sum_{i=1}^{n}\left(\frac{\partial f(X)}{\partial X_i}\Big\vert_{\mu}\right)^3 \mu_i^{(3)}$
G_{41}	$\mu_C^{(4)} - 3(\mu_C^{(2)})^2$	$\sum_{i=1}^{n}\left(\frac{\partial f(X)}{\partial X_i}\Big\vert_{\mu}\right)^4 \left[\mu_i^{(4)} - 3(\mu_i^{(2)})^2\right]$

Table 6. Additive Moments for SWBS 150

$$W_{150} = V_{ss} \cdot \rho_{ss}$$

$$G_{11} = \mu_{W150} = \mu_{V_{ss}} \cdot \mu_{\rho_{ss}}$$

$$G_{21} = \mu^{(2)}_{W150} = (\mu_{\rho_{ss}})^2 \cdot \mu^{(2)}_{V_{ss}} + (\mu_{V_{ss}})^2 \cdot \mu^{(2)}_{\rho_{ss}}$$

$$G_{31} = \mu^{(3)}_{W150} = (\mu_{\rho_{ss}})^3 \cdot \mu^{(3)}_{V_{ss}} + (\mu_{V_{ss}})^3 \cdot \mu^{(3)}_{\rho_{ss}}$$

$$G_{41} = \mu^{(4)}_{W150} = (\mu_{\rho_{ss}})^4 \,(\mu^{(4)}_{V_{ss}} - 3(\mu^{(2)}_{V_{ss}})^2) + (\mu_{V_{ss}})^4 \,(\mu^{(4)}_{\rho_{ss}} - 3(\mu^{(2)}_{\rho_{ss}})^2)$$

which was chosen for this thesis has four parameters; alpha, beta, a (low value), and b (range). The first four moments can be expressed in terms of these parameters as shown in Table 7. Expressing the parameters in terms of the moments is considerably more complicated. McNichols used a lookup table in his dissertation but Wilder and Black [32] derived a closed form solution which is given in Appendix B.

Returning to the case of the error due to regression analysis, it can be handled by estimating the moments from the data as shown in Table 8. If for a particular estimating relationship there is no input uncertainty, the only moments are those due to the regression. If both input and regression uncertainty exist, the moments add together [33].

The next chapter discusses the implementation of these methods in a ship synthesis model.

Table 7. Moments in Terms of Beta Parameters

Central Moments (Two-Parameter Beta)	Generalized Additive Moments (Four-Parameter Beta)
$\mu_S = \frac{\alpha+1}{\alpha+\beta+2}$	$G_1 = a + b\mu_S$
$\mu_S^{(2)} = \sigma^2 = \frac{(\alpha+1)(\beta+1)}{(\alpha+\beta+3)(\alpha+\beta+2)^2}$	$G_2 = b^2\mu_S^{(2)}$
$\mu_S^{(3)} = \frac{2(\alpha+1)(\beta+1)(\beta-\alpha)}{(\alpha+\beta+4)(\alpha+\beta+3)(\alpha+\beta+2)^3}$	$G_3 = b^3\mu_S^{(3)}$
$\mu_S^{(4)} = \frac{3(\alpha+1)(\beta+1)[(\alpha+1)(\beta+1)(\alpha+\beta-4) + 2(\alpha+\beta+2)^2]}{(\alpha+\beta+5)(\alpha+\beta+4)(\alpha+\beta+3)(\alpha+\beta+2)^4}$	$G_4 = b^4[\mu_S^{(4)} - 3(\mu_S^{(2)})^2]$

Table 8. Estimating Moments for Regression Relationships

$$\mu^{(2)} = f(x)^2\ 1/(n-2)\ \Sigma[(y_i - (a_1x_i + a_o))/f(x_i)]^2$$

$$\mu^{(3)} = f(x)^3\ 1/(n-2)\ \Sigma[(y_i - (a_1x_i + a_o))/f(x_i)]^3$$

$$\mu^{(4)} = f(x)^4\ 1/(n-2)\ \Sigma[(y_i - (a_1x_i + a_o))/f(x_i)]^4$$

Where $f(x)$ is the heteroscedastic relationship

CHAPTER 5

IMPLEMENTATION OF UNCERTAINTY METHODOLOGY IN A SYNTHESIS MODEL

As discussed in the previous chapter, analytic means exist to calculate probability distributions for estimating relationships. This chapter will propose a system of implementing these methods in an actual synthesis model.

The Monohull Surface Combatant (MONOSC) configuration of ASSET (Advanced Surface Ship Evaluation Tool), maintained by the David Taylor Ship Research and Development Center (DTNSRDC) was chosen for use in this thesis for several reasons:

(1) ASSET is extremely flexible in terms of being able to handle new technology.

(2) ASSET is a modular program which means that implementation of the proposed methodology can be done in an incremental fashion.

(3) The logic and estimating relationships for ASSET are well documented.

(4) ASSET was available at MIT on the 13A Ships Computer Aided Design System (SCADS).

5.1 Description of ASSET

ASSET [15] is actually an "umbrella" program with applicability to monohull surface combatants, planing craft, hydrofoils, and small waterplane area twin hull (SWATH) type ships. It is planned to extend this in the

future to air capable monohulls, auxiliary monohulls, air cushion vehicles and surface effect ships. ASSET achieves this wide range by its modular structure. Common modules are used whenever possible, and configuration dependent modules are used to cover unique aspects of a particular ship type.

Figure 11 shows an overall view of the ASSET program. The user controls ASSET with an interactive executive program. A data bank is maintained with information for several complete ships and individual components. The computational modules use data from and modify a current model that has been selected from the data bank. Each of the computational modules offer several possible screens of information to the user in both tabular and graphical format.

Figure 12 shows a flow chart of the computational modules for the MONOSC configuration of ASSET. A brief overview [34] of each of modules follows.

(a) The Initialization module uses simplified parametric methods to check the input data for consistency and to make initial estimates of the basic design parameters.

(b) The Hull Geometry module calculates hull form characteristics based on an input set of offsets and can also modify the size or shape of the hull.

(c) The Hull Structure module calculates scantling data for the hull based on either calculated loads or on input by the designer. At the present stage of development, it does not optimise the structure for either weight or cost.

(d) The Resistance module calculates ship drag using the

Figure 11. ASSET System Concept

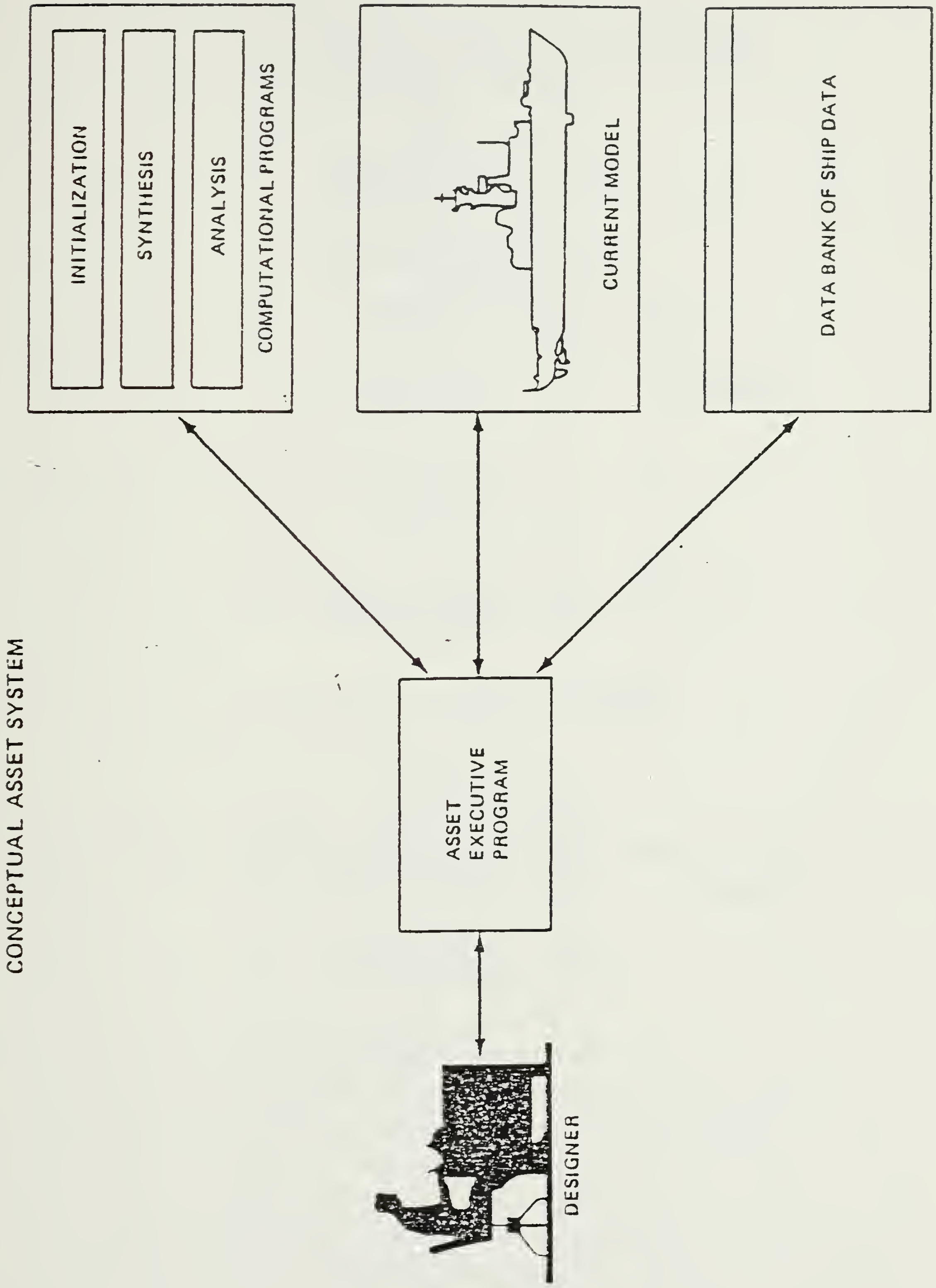
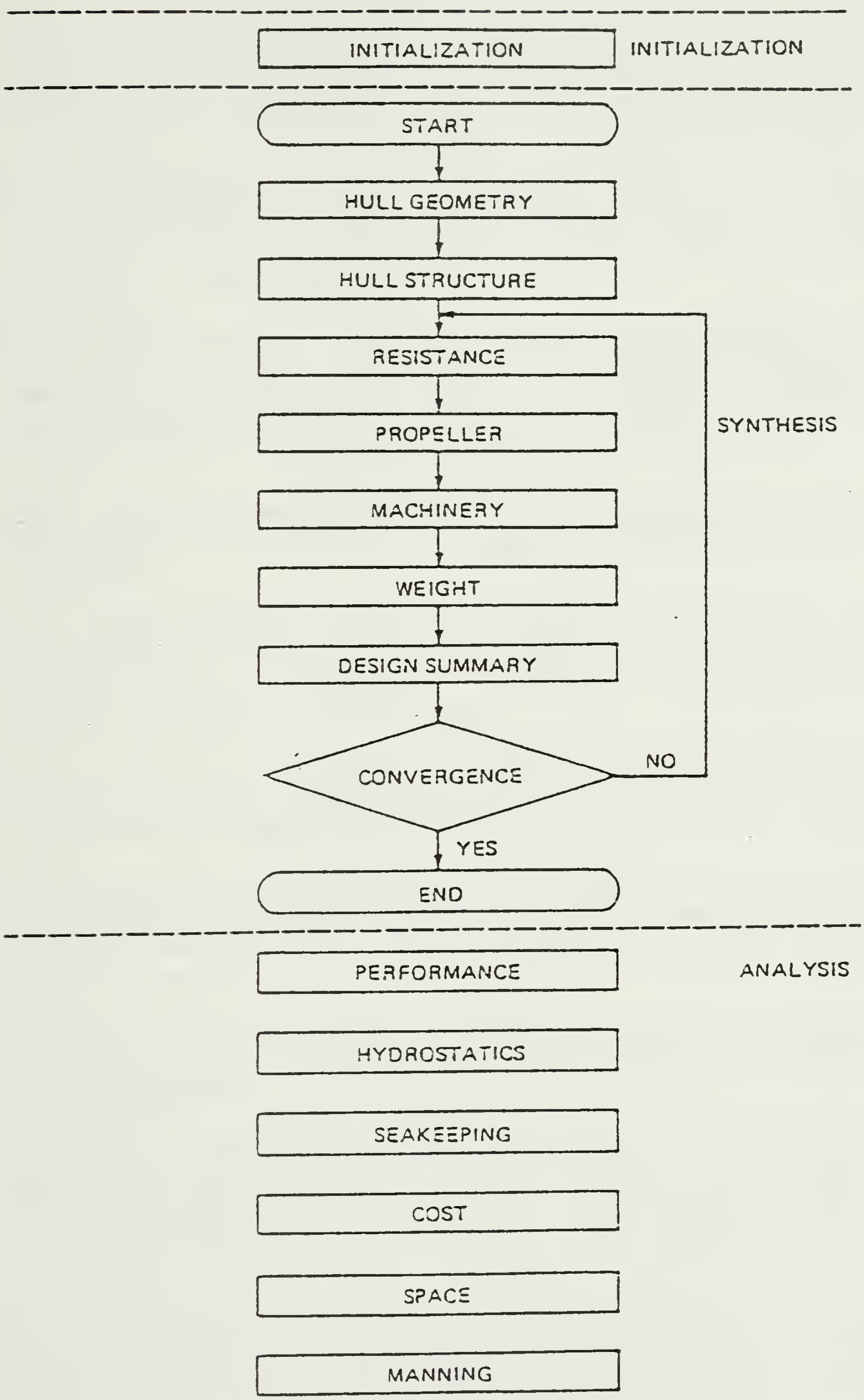


Figure 12. Flowchart for MONOSC ASSET



Taylor series data modified by a user input worm curve and either an ATTC or ITTC friction line.

(e) The Propeller module determines the geometry of the ship propeller and the shaft power needed for the endurance and sustained speeds. The module can use either user input propeller data, the Troost-B series, or an analytic design based on lifting line theory.

(f) The Machinery module computes electric power requirements and sizes the machinery if it is not fixed by the user. The module also calculates endurance fuel requirements.

(g) The Weight module calculates a detailed weight and center of gravity breakdown for the ship using the Navy Ship Work Breakdown Structure, SWBS. The algorithms are largely based on those used in the NAVSEA DD08 synthesis model.

(h) The Design Summary module provides selected data from the previous six modules (Items b. through g.) for the designer.

(i) The Performance Analysis module calculates the degredation in performance of a complete, synthisized ship caused by hull fouling, machinery plant deterioration and sea state.

(j) The Hydrostatic Analysis module calculates hydrostatic properties of form, floodable length, intact stability and damaged stability.

(k) The Seakeeping Analysis module calculates the Bales rank factor for the hull form. This ranking is on a scale from one to ten and considers pitch and heave motions only.

(l) The Cost Analysis module estimates unit production and life cycle costs using various parametric relationships.

(m) The Space Analysis module calculates the total volume and area requirements for the ship using the Navy Ship Classification System (SSCS). The algorithms are largely based on those used in the NAVSEA DD08 synthesis model.

(n) The Manning Analysis module estimates the number of officers, chief petty officers and enlisted personnel required to man the ship and the total man-hours required to accomplish required ship tasks.

As can be seen in Figure 12, the computational modules are classified as being either Initialization, Synthesis or Analysis. The Initialization module uses simplified estimating relationships in order to establish a starting point for further work. Both the Synthesis and Analysis type modules use detailed analytical or parametric techniques with the difference that the output from a Synthesis module modifies the current model, while the Analysis type just provide additional information.

As previously mentioned, the user controls the execution of each of these modules. An automatic iterative loop can be invoked as shown in Figure 12 which balances the energy and weight for a fixed hull geometry and structure. At the present stage of development, the user must manually balance the space and stability requirements by using analysis modules to determine changes required and then modifying the current model.

5.2 Proposed Implementation Method

Implementing the uncertainty methods described in Chapter 4 in a synthesis model is different from previous costing applications described in [1], [32] and [35] because of the iterative nature of the synthesis process. Because the uncertainty methodology add more computations, it is desirable to reduce the number of iterations required to produce a converged solution. Finally, because of the new approach being taken and because new regression

algorithms must be derived, it was considered desirable to take advantage of ASSET's modularity and have a plan that would allow the methodology to be implemented in an incremental manner.

Examining the four areas required to be balanced (Energy, Space, Weight and Stability) the following strategy for implementation is recommended.

(a) Energy: This is a low priority area. The existing resistance module in the synthesis loop should be retained and use mean values of the parameters. The power installed and the fuel weight should be fixed and treated as certain values. Likewise the electric generating plant should be fixed in size, but a pdf for electric loads generated for informational purposes. The best place to implement the uncertainty methodology is in the performance analysis module to calculate distributions for sustained speed achieved and range at endurance speed.

(b) Space: The geometry of the ship should be held constant as in the current ASSET model. ASSET calculates space available with a high degree of accuracy so it should be treated as a certain value. The space analysis module should calculate a probability distribution for the space required and the designer then provided with a graph showing the probability of the space available being greater than or equal to the space required. Two options should then be available to the designer. The first, to be used primarily for technology assessment and early stage design, would use the mean of the space required to balance the ship and the hull size fixed. Then the size of the deckhouse would be considered variable with a pdf derived from the space required pdf. Current synthesis models also change deckhouse size first because the overall impact on the ship is normally less. Volume would then be an uncertain quantity for those weight estimating relationships that use volume as an input. The second option would be used during later stage design when the size of the ship must be fixed. This option would have the designer use the space required probability distribution as a guide to fixing the size of the ship and then would use volume available for weight estimating relationships with no input uncertainty.

(c) Weight: Changes required are primarily in the weight module. To reduce the computational effort, a design converged by the current methodology should be used as a starting point. If the first order Taylor Series approximation (see Table 5) is used, the mean of the function is equal to the function of the means and only one pass through the module is required. As discussed in part (b), volume could either be a certain or uncertain input, depending on the design phase.

(d) Stability: Balance should be checked by a GM/B criteria. GM will be uncertain due to the vertical center of gravity, which is calculated in the weight module. The position of the metacenter is calculated exactly in the hull geometry module. In the design summary module, the designer should be given the probability that the GM/B is within a given range and the mean value of GM/B.

5.3 Demonstration Module

The full implementation of this methodology was beyond the scope of this thesis. However, for demonstration purposes it was decided to write a replacement weight module for ASSET that would show the usefulness of the proposed methodology. This module was designed to return the parameters for a Beta pdf for full load displacement, light ship weight, and the one digit level SWBS weight groups. An auxiliary program would then plot both the pdf and the cumulative distribution function.

The module was written using the LOTUS 1-2-3 spreadsheet program running on a Zenith Z-100 microcomputer. The procedure for using it is to achieve a converged ship on the mainframe ASSET program. The parameters from the current model needed for the spreadsheet program are then manually entered and the

spreadsheet automatically calculates the pdf parameters.

Several assumptions were made for this demonstration tool. First, the existing ASSET algorithms were used as a basis because; (1) the raw data was not available to derive new ones using the methodology from Section 4.1 and (2) time would not have permitted it. Since the statistical data was not available either, a default assumption was made in all estimating relationships with regression analysis uncertainty that the distribution was; symmetric, had a range equal to 20% of the mean, and that the limits were plus or minus three standard deviations. This gave a psuedo-Gaussian distribution that assumed the estimating relationship was accurate to plus or minus 10%

The number of options in the input was cut down since the module was intended for demonstration purposes only. The input options can be seen in the example study printout in Appendix C. Also, as mentioned in the previous chapter, dependency between estimating relationships was not considered in this thesis. The author believes this issue must be approached with care. Although many relationships share the same input variable (e.g. Volume is used by many relationships in the auxiliaries and outfitting areas), it should be kept in mind that most of the relationships are derived from regression analysis and the choice of input variable is based on what is available in early stage design. Therefore the dependencies are probably weak.

One dependent item that was not adequately treated was Hull Structural Weight, SWBS Groups 110-140. ASSET calculates this weight as a function of the hull geometry and scantlings designed in the Hull Structure module. The spreadsheet program currently takes these weights from ASSET and calculates a distribution about that mean. In actuality, the variation in full-load displacement causes a variation in loads, causing uncertainty in scantlings and thus in hull weight. This area needs further research before this methodology can be fully implemented.

The next chapter will discuss an example tradeoff study using this module and better illustrate the usefulness of the proposed methodology to a decisionmaker.

CHAPTER 6

CASE STUDY USING UNCERTAINTY METHODS

This chapter presents a tradeoff study designed to illustrate the usefulness of the proposed methodology to a decision maker. The probability distributions shown for weight were calculated using the LOTUS modules described in the previous chapter. The actual numbers are not accurate due to the shortcomings of the present model, but they are sufficient for this pedagogical purpose. The distributions for cost, sustained speed, and range were assumed but were designed to be realistic.

6.1 Input to Tradeoff Study

The case study presented here was based on an actual tradeoff analysis conducted for the DDG-51 design. The issue was whether to use a controllable-reversible pitch propeller (CRPP) as on previous designs, or whether to develop a reversing reduction gear (RRG) which would allow the use of a fixed pitch propeller. Advantages seen for the RRG were a lower total system weight leading to a smaller ship, and greater propulsive efficiency leading to a greater sustained speed and reduced fuel costs. The disadvantages of the RRG were research and development costs and risk.

For the purposes of this thesis, the example study was conducted using the ASW frigate developed in Goddard's

thesis [17] as a baseline. The study was considered to be a general technology assessment, not linked to a particular ship acquisition project.

The study was begun by modifying Goddard's baseline in ASSET, which had electric transmission, into two variants, one with CRPP's and one with RRG's. The only difference between the designs at this point was the transmission and propeller systems. The designs were then balanced using the conventional criteria discussed in Section 3.1. The results of this portion of the study are presented in Table 9.

At this point, the analysis of the uncertainty in the weight estimates could be conducted using the LOTUS module. The necessary parameters from ASSET were extracted from the databank and then input into the LOTUS weight module for each ship. This data can be found in Appendix C. For this initial study, the number of input items with uncertainty was reduced to three, in order to best see the relative impacts of the two technologies. These items were: superstructure volume, superstructure density, and gear K factor.

The K factor was chosen because it was an input variable to the estimating relationship for SWBS weight group 241, Propulsion Reduction Gears, and would have a direct impact on the weight of that subsystem, since the estimating relationship was different for a conventional

Table 9. Results of ASSET Tradeoff Study

	CRPP	RRG
FULL LOAD WT	5992 LTONS	5883 LTONS
LT SHIP WT	4476 LTONS	4412 LTONS
FUEL WT	1080 LTONS	1037 LTONS
LBP	437 FT	435 FT
BEAM	51.4 FT	51.2 FT
DRAFT	19.3 FT	19.2 FT
DESIGN SPEED	27.6 KTS	28.1 KTS
ELECT LOAD	4281 KW	4264 KW

reduction gear and a reversing one. The same pdf was assumed for both ships and can be seen in Figure 13.

Deckhouse density is included as an input item because ASSET allows several choices of material for the superstructure. In actuality though, its value is derived by regression analysis so it should have a probability distribution. Since the value was independent of the technology being evaluated, the same pdf, shown in Figure 14, was used for both ships.

Finally, the deckhouse volume was assumed to vary as a measure of the total volume required. Both designs had uncertainty in volume required due to regression uncertainty as well as the transmission technology. As explained in the last chapter, for a technology type study the hull size should be fixed with the superstructure varying to meet the space requirement. If an item is displaced in the hull by increased machinery volume requirements for instance, it can be relocated to the deckhouse.

The mean values for superstructure volume were based on the ASSET calculations with assumptions made for the variance, skew and kurtosis. The RRG ship was assumed to have a greater amount of variance and skew because of greater uncertainty for the volume requirements of the new reversing gear. The pdf's for superstructure volume are shown in Figure 15.

Figure 13. PDF for Gear K Factor

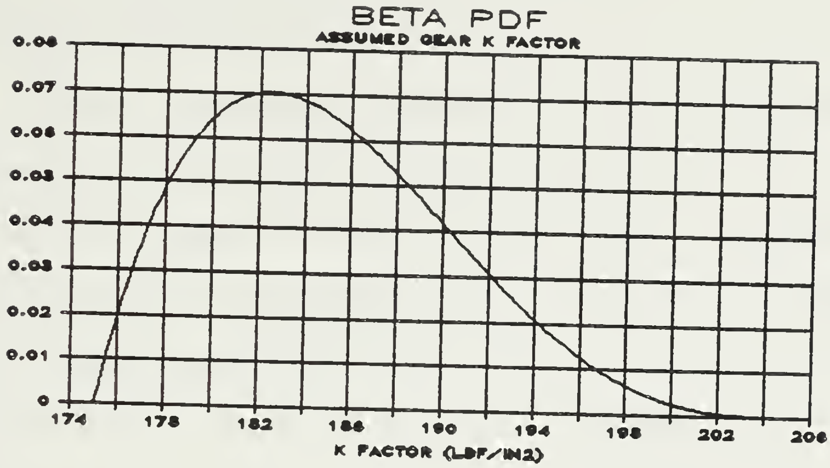


Figure 14. PDF for Deckhouse Density

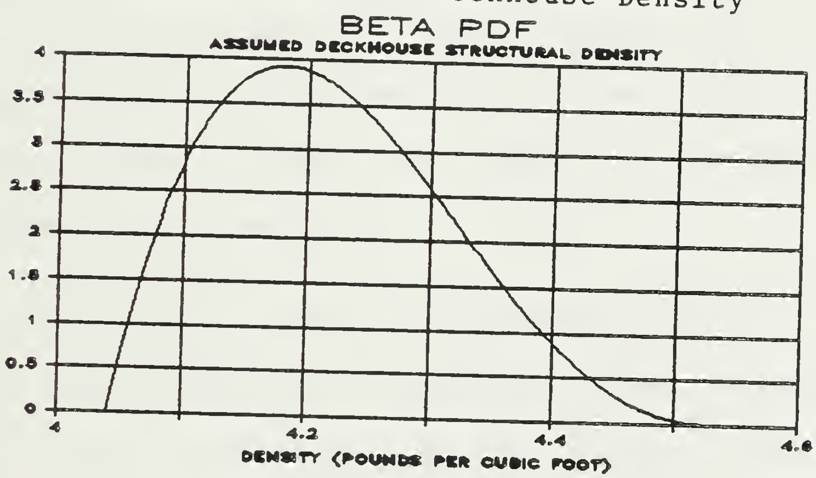
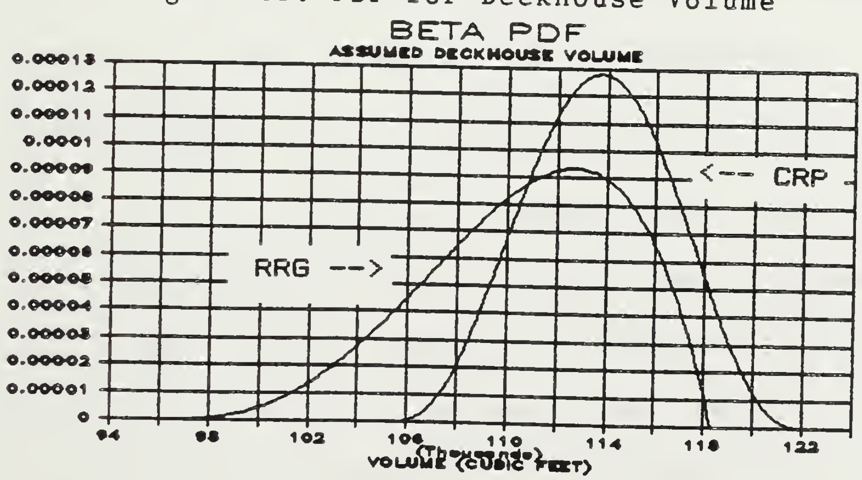


Figure 15. PDF for Deckhouse Volume



6.2 Samole Presentation

After a tradeoff analysis or technology assessment is conducted, the results must be presented to a decision maker who will make a choice. The whole purpose of this approach to risk assessment is to provide a quantitative, clearer classification of the risk to the decision maker. If this new procedure is to be accepted, this advantage must be demonstrated.

The following is an example of how the information from this tradeoff study would be presented to a decision maker. It uses probability distributions for weight that were generated by the LOTUS analysis and distributions for acquisition cost, sustained speed, and range that were assumed by the author.

> "Good morning Admiral. The purpose of this meeting is to make a decision concerning the development of a reversing reduction gear for future combatant ships. The possible decisions are to provide funding for full scale development, to continue exploratory development, or to discontinue development.
>
> The first vu-graph (Figure 16) shows the nominal characteristics of our technology assessment frigate equipped with reversing reduction gears and fixed pitch propellers versus one with the baseline controllable-reversible pitch propellers.

Figure 16. Nominal Characteristics for CRPP and RRG Ships

	CRPP	RRG
LBP	437 FT	435 FT
BEAM	51.4 FT	51.2 FT
DRAFT	19.3 FT	19.2 FT
SHP INSTALL	52,500 HP	52,500 HP
KW INSTALL	8000 KW	8000 KW
ENDURANCE	4500 NM	4500 NM
PAYLOAD	970 LTONS	970 LTONS
CREW SIZE	301	301

DISPLACEMENT: This vu-graph (Figure 17) shows the results of the analysis of impact on full load displacement. The upper curve shows the probability that the displacement will be equal to or less than the value on the x-axis. As can be seen, for any given level of probability, the RRG ship has an approximately 100 ton advantage. The probability density curve at the bottom of the chart shows that in this particular case the distributions exhibit little skew. Also, from the height of the modes, it can be seen that the RRG ship has slightly more variance. The variance for the baseline with CRPP's is due primarily to uncertainty in the regression algorithms.

Examining the distributions for Group 100 (Figure 18) and Group 200 (Figure 19) weights, the source of this difference in variance is primarily due to structural weight. Analysis showed that this is due to greater uncertainty in the volume requirements of the reversing gear.

BOTTOM LINE: RRG has same weight risk as CRPP with 100 ton advantage.

ACQUISITION COST: Considering relative acquisition cost next, this vu-graph (Figure 20) shows that there is a 65 percent chance the acquisition cost of the RRG ship will be less than that of the CRPP design. The increased variance and skew for the RRG ship is caused

Figure 17. CDF and PDF for Full Load Displacement

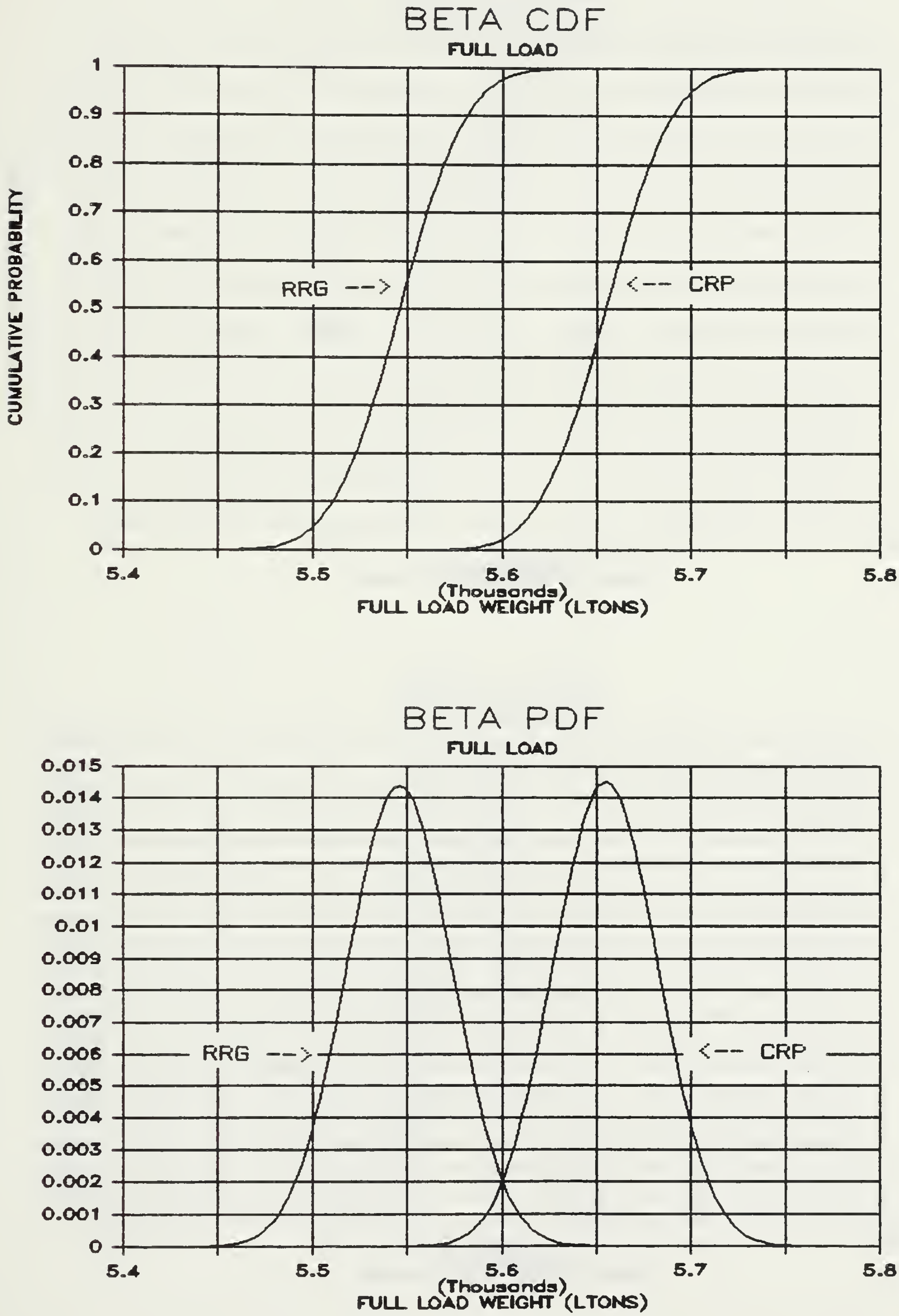
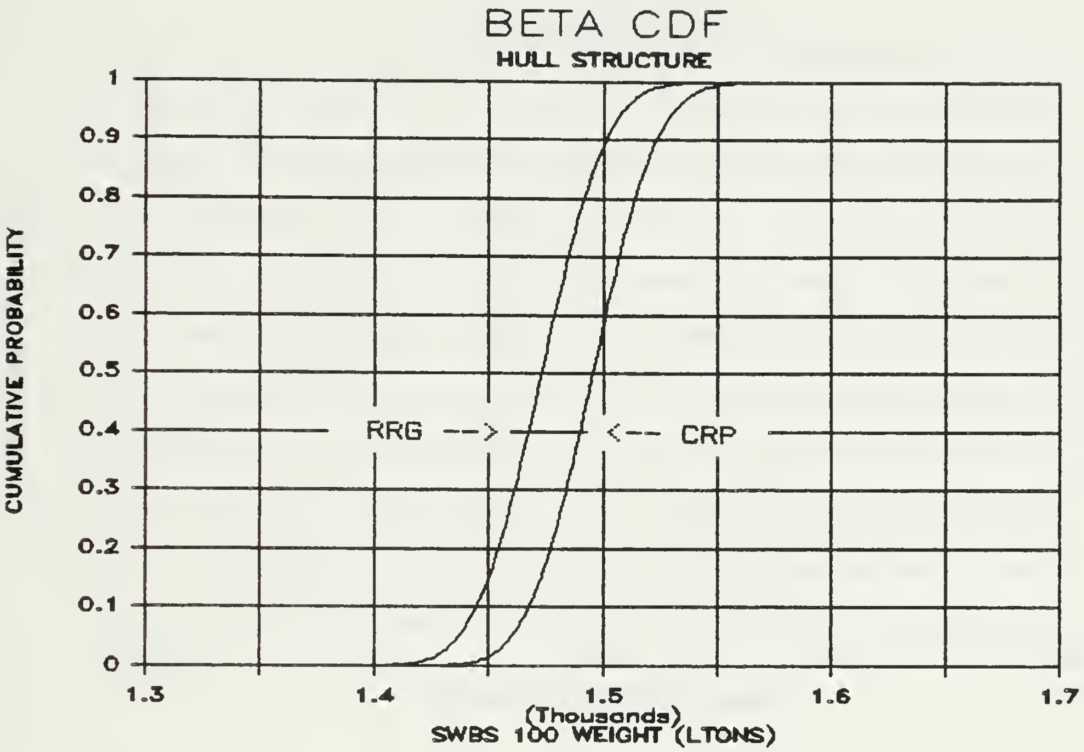


Figure 18. CDF and PDF for SWBS Group 100



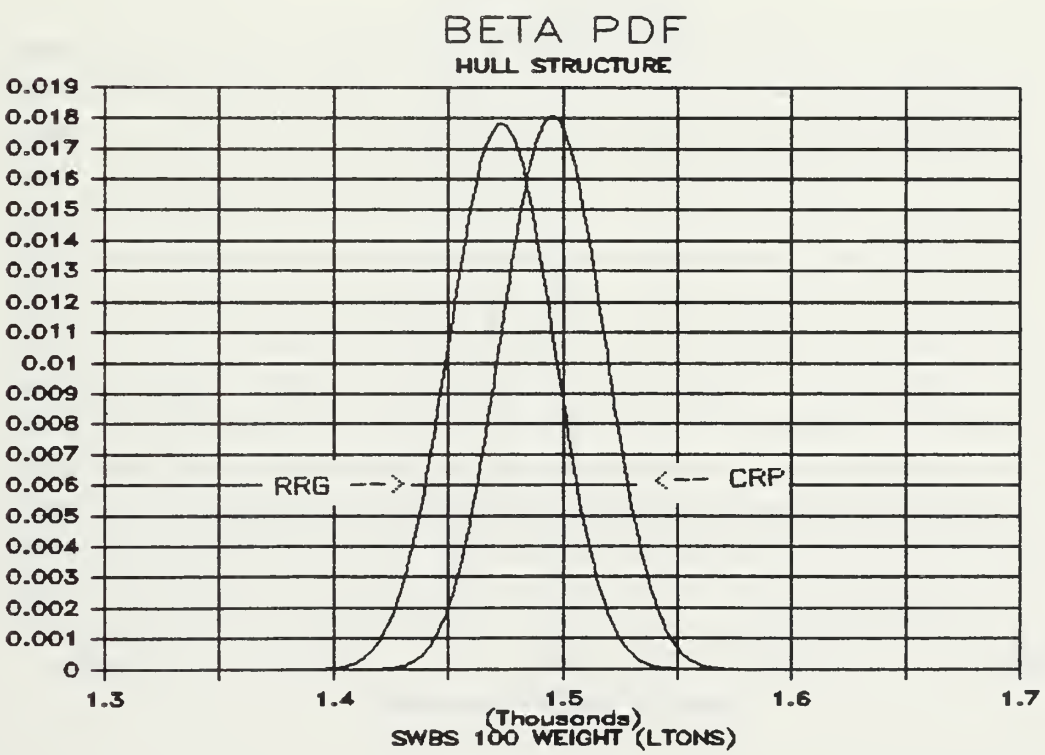
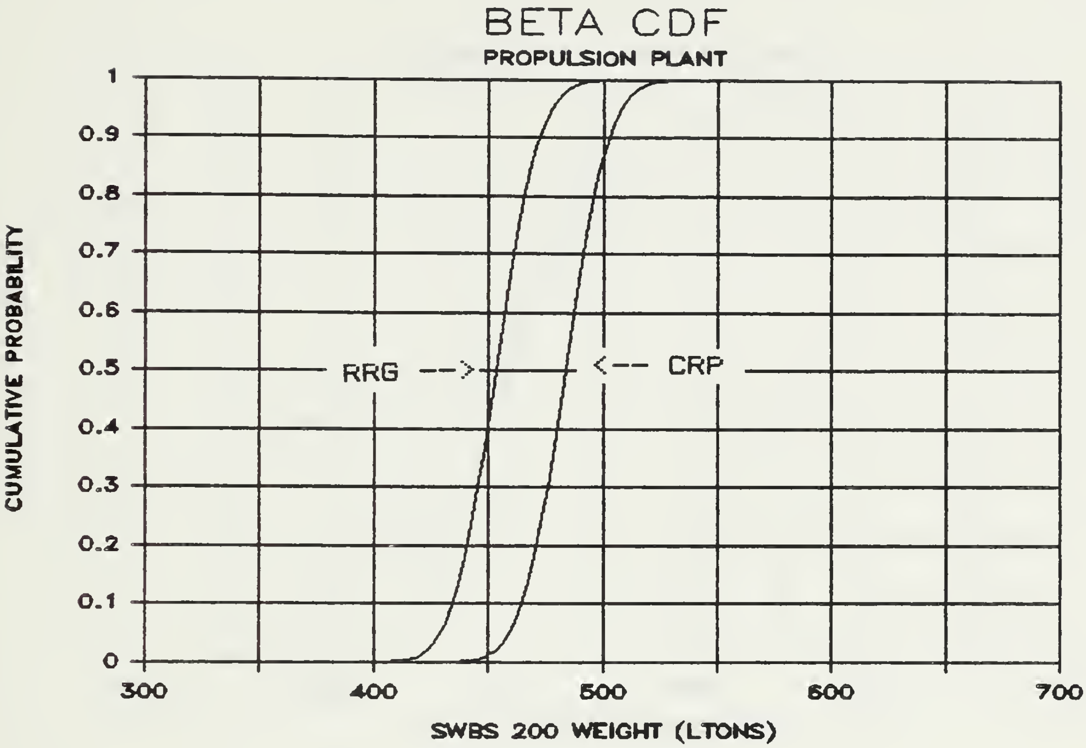


Figure 19. CDF and PDF for SWBS Group 200



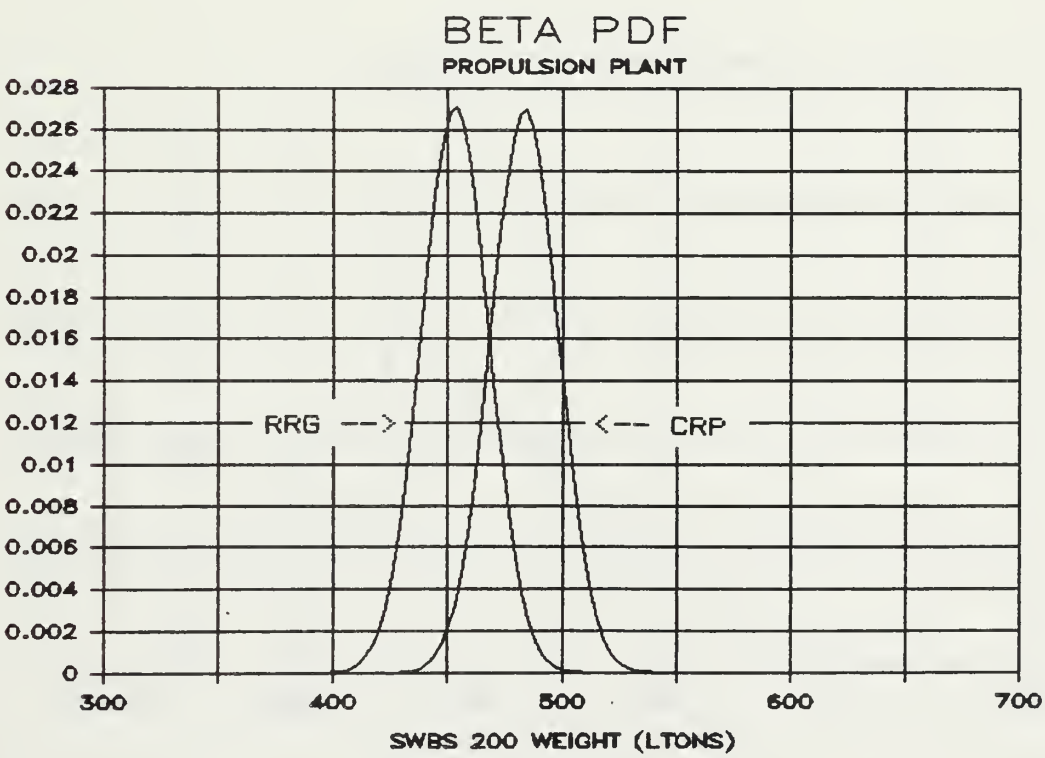
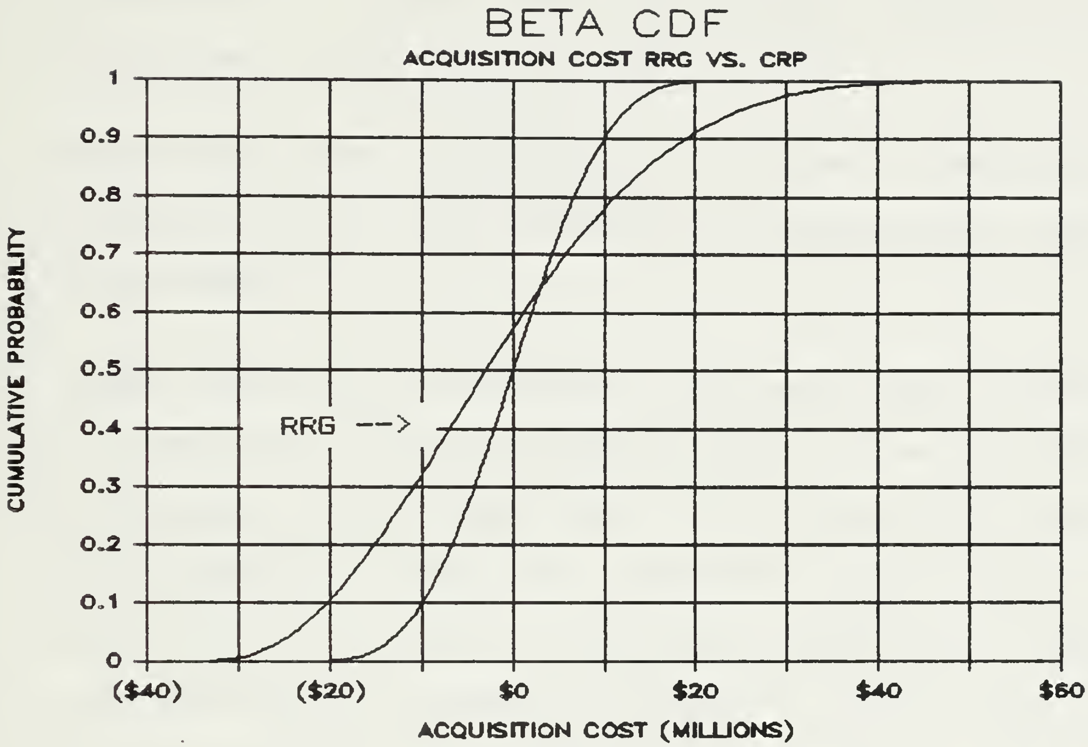
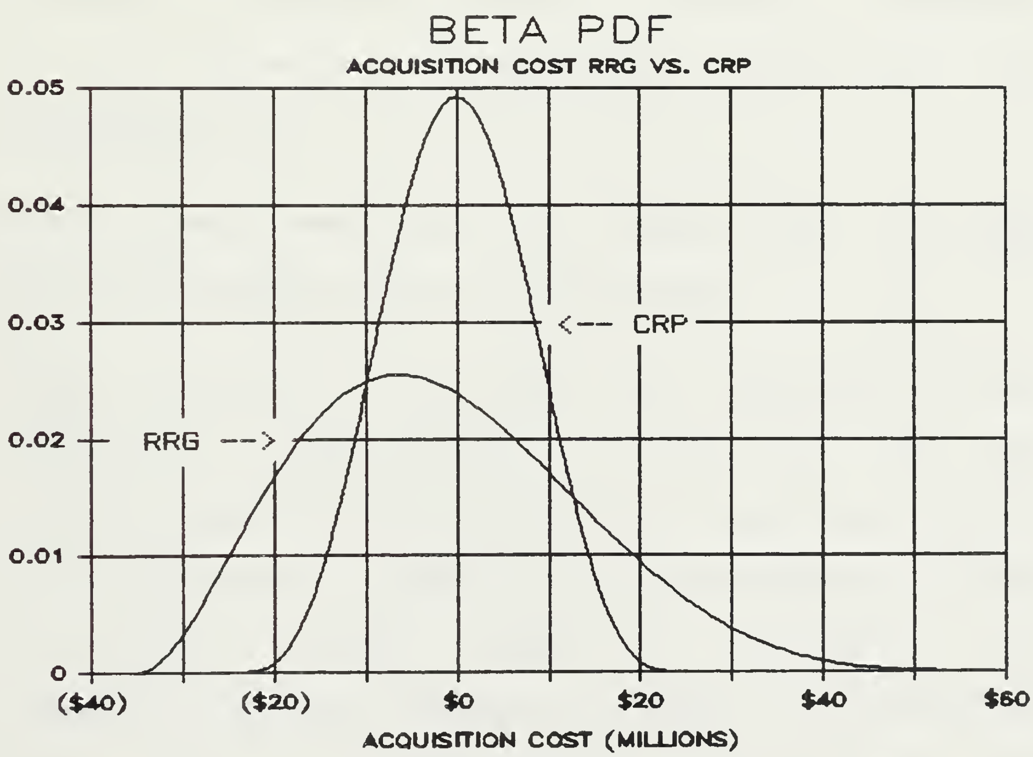


Figure 20. CDF and PDF for Acquisition Cost





primarily by the possibility of developmental problems resulting in higher than anticipated research and development costs.

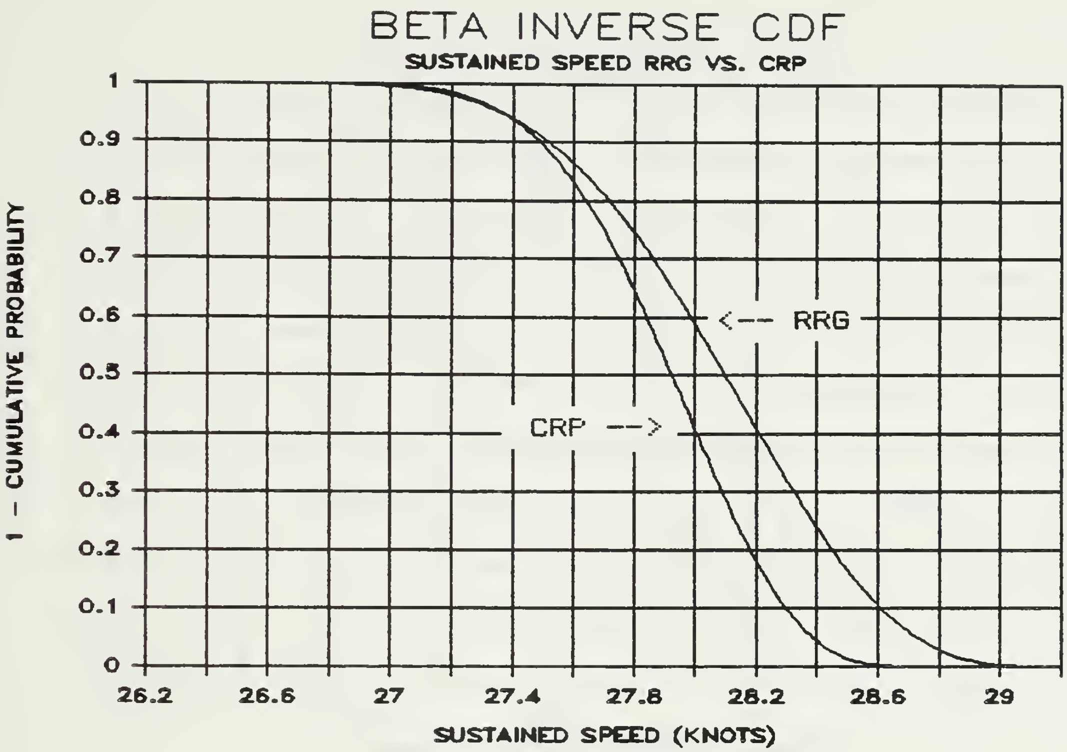
BOTTOM LINE: RRG has medium cost risk due to potential setbacks in R & D. Has 65% chance of being better than CRPP though.

SPEED: Concerning performance, this next chart (Figure 21) shows the expected sustained speed given a fixed powerplant of two LM2500-30's. The cumulative curve here shows the probability that the speed will be *greater* than or equal to the value. The RRG ship shows an advantage for probability levels of less than 90 percent.The RRG design has greater uncertainty as can be seen from the pdf, because of greater uncertainty in the estimates for propulsive coefficient and appendage drag.

BOTTOM LINE: RRG ship will always have a speed advantage ranging from nil to 0.4 Kts.

RANGE: Similarly, the curve for endurance range (Figure 22) shows greater variance for the same reasons. These curves indicate that given the amount of fuel assumed, the RRG ship has an 80 percent chance of having a greater range. If necessary, this probability can be increased by adding a margin for fuel, which will cause the displacement and acquisition cost curves to shift to the right and the

Figure 21. CDF and PDF for Sustained Speed



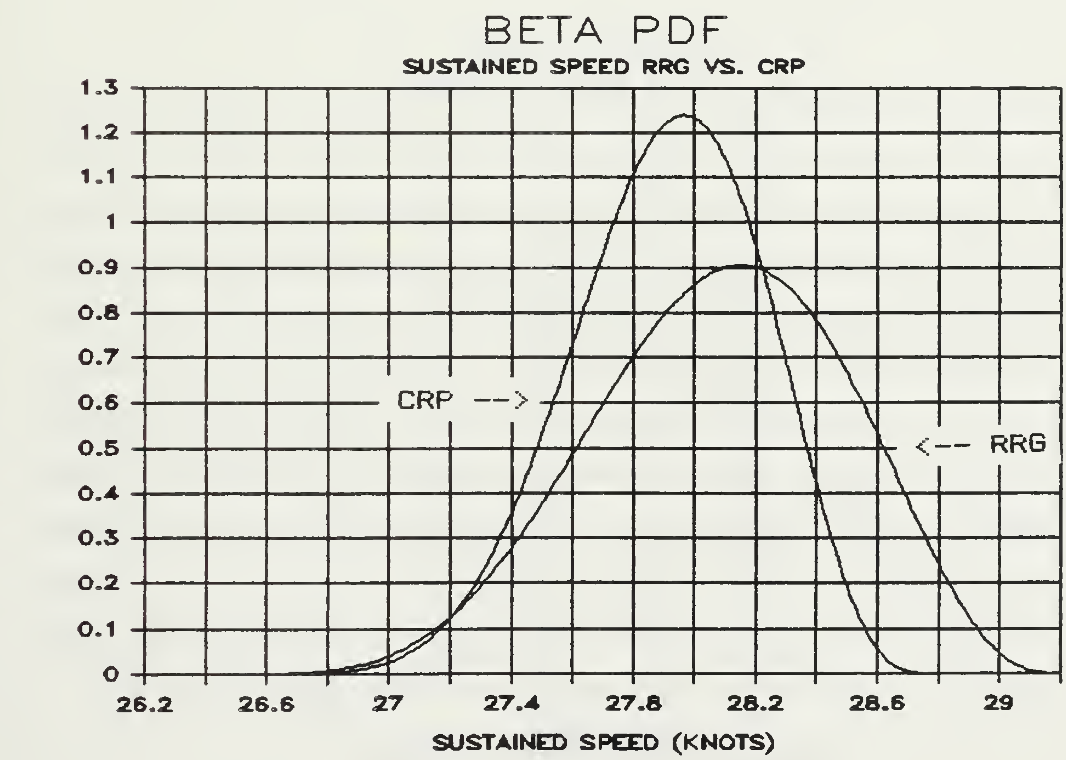
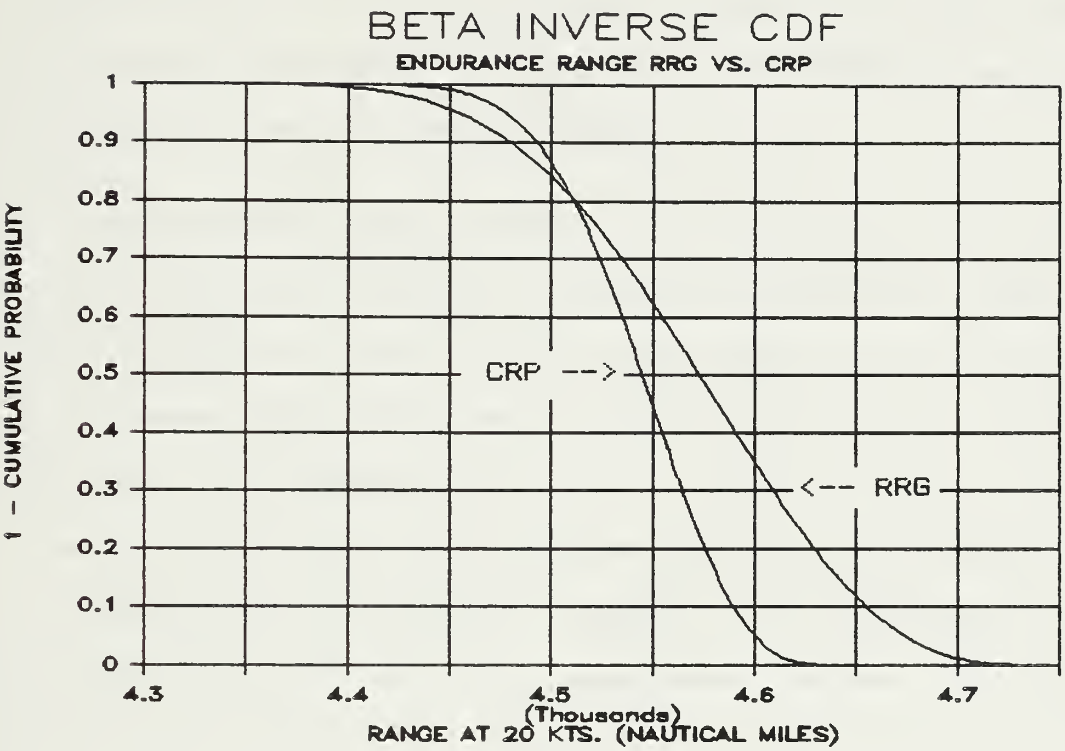
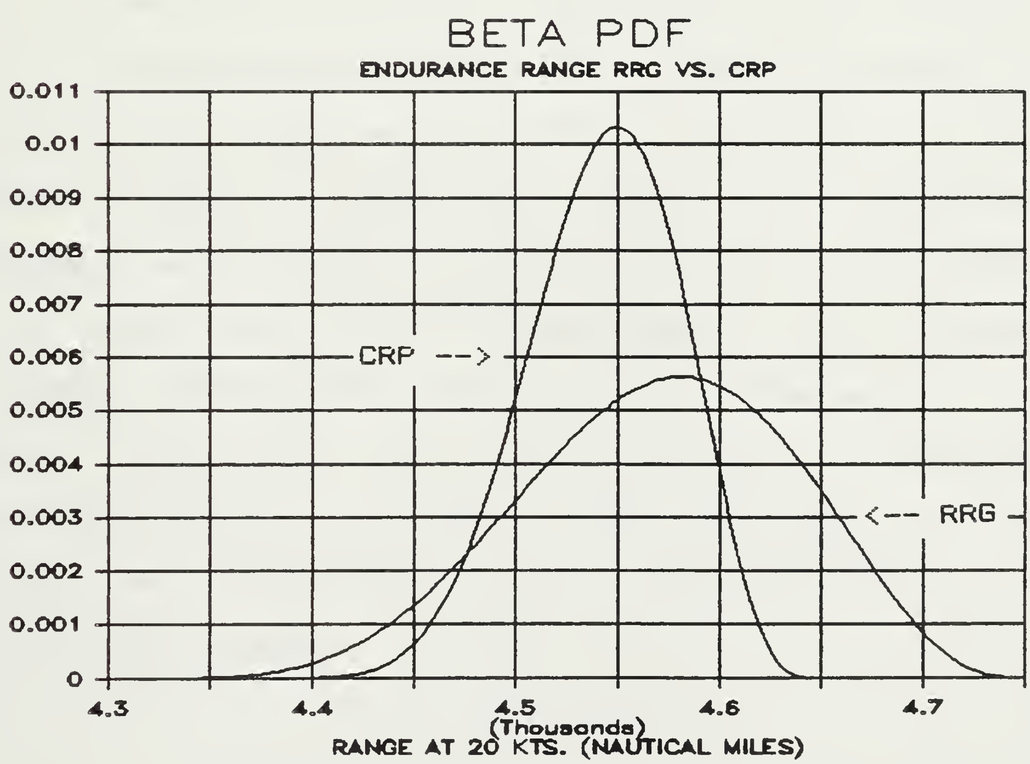


Figure 22. CDF and PDF for Endurance





sustained speed to shift to the left.

BOTTOM LINE: RRG has greater uncertainty on range attained but based on current fuel amount has 80% chance of having greater range.

OVERALL BOTTOM LINE: Based on this analysis, we rate the technical risk of the reversing reduction gear to be low and the cost risk to be medium. We therefore recommend that development be continued at the current level, with emphasis on the volumetric requirements and those elements causing the greatest uncertainty in research and development costs. This approach will allow us to refine this assessment before the next decision point in six months."

From this example, the advantages of this approach to risk assessment can be seen. The graphical display of information gives the decision maker a better feel for the risk and possible consequences without the facts being obscured by margins. The source of uncertainty and whether it is caused by the new technology or by standard estimating relationships can also be determined. This example concentrated on the technology assessment application, however this approach has advantages for other phases of ship design as well which will be discussed in the concluding chapter.

CHAPTER 7

CONCLUSIONS AND RECOMMENDATIONS

The purpose of this thesis was to show the usefulness of using probability distributions to classify risk in the naval ship design process. Additionally, it was necessary to demonstrate how these probability distributions could be analytically generated.

The example study in the previous chapter demonstrated the usefulness of this methodology for the technology assessment phase of the design process. The author believes that this approach would also be useful for the purpose of establishing, monitoring and managing the margin policy during ship design and construction. The margin could be established using the cumulative distribution curve for a desired level of probability. Those items identified as having the greatest variance and impact could then be monitored more closely to avoid exceeding the margin. Also, as more detailed information came in, the probability distributions could be updated to show the potential for exceeding or beating the margin.

This approach also has a beneficial effect by causing the subsystem designers to consider the possible spread of values for their subsystem. Perhaps more importantly, it helps to remove the uncertainty that the

ship designer might have as to whether the subsystem designer was giving the best possible case in his input, or conversely, had added margin so as to be more conservative.

The following steps are recommended to better assess and manage risk in the ship design process and to implement the proposed methodology:

(1) Educate both subsystem and ship designers in the risk assessment methodologies described in this thesis.

(2) Require subsystem designers to submit a probability density function as part of the technology characterization process and indicate how it was derived. The important point here is not the exact methodology used, but the thought process behind it (i.e. an assessment based on good engineering judgement would be better than analytical methods poorly applied).

(3) Implement the method of moments methodology in a program for monitoring weight during detailed design and construction.

(4) Reexamine the present synthesis model estimating relationships using the assumption of heteroscedasticity and evaluate the variance, skew and kurtosis.

(5) Conduct further research on implementing the proposed methodology in the ASSET synthesis model. Specific areas requiring further work are:

   (a) accounting for the impact on structural weight of variations in full load displacement.

   (b) establishing need to consider dependency between estimating relationships.

These steps are listed in an ascending order of complexity and logical order of implementation. It is realized that this methodology adds more complexity to the design process but it is believed that it would add clarity to the important issue of risk and help in making the proper decisions.

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APPENDIX A

LEAST SQUARES DERIVATIONS

A.1 Ordinary Least Squares

Assumed Form of Equation

$$y_i = a_1x_i + a_o + \varepsilon_i$$

Aim is to minimize sum $\Sigma\varepsilon_i^2 = \Sigma(y_i - a_1x_i - a_o)^2$

Will derive two equations in two unknowns

$$\partial\Sigma\varepsilon_i^2/\partial a_1 = 2\Sigma(y_i - a_1x_i - a_o)(-x_i) = 0$$

$$= \Sigma(y_ix_i - a_1x_i^2 - a_ox_i)$$

$$= \Sigma(y_ix_i - a_1\Sigma x_i^2 - a_o\Sigma x_i \implies$$

$$a_1\Sigma x_i^2 + a_o\Sigma x_i = \Sigma y_ix_i$$ First Equation

$$\partial\Sigma\varepsilon_i^2/\partial a_o = 2\Sigma(y_i - a_1x_i - a_o)(-1) = 0$$

$$= \Sigma y_i - a_1\Sigma x_i - a_on \implies$$

$$a_1\Sigma x_i + a_on = \Sigma y_i$$ Second Equation

A.2 Weighted Least Squares (Heteroscedasticity)

Assume Variance $\{y/x\} = \sigma^2f^2(x)$

Equation becomes:

$$y_i/f(x_i) = a_1x_i/f(x_i) + a_o/f(x_i) + \varepsilon_i$$

$$\Sigma\varepsilon_i^2 = \Sigma(y_i/f(x_i) - a_1x_i/f(x_i) - a_o/f(x_i))^2$$

Again deriving two equations in two unknowns:

$$\partial\Sigma\varepsilon_i^2/\partial a_1 = 2\Sigma(y_i/f(x_i) - a_1x_i/f(x_i) - a_o/f(x_i))(-x_i/f(x_i)) = 0$$

$$= \Sigma (y_i x_i / f^2(x_i) - a_1 x_i^2 / f^2(x_i) - a_0 x_i / f^2(x_i))$$
==>

$$a_1 \Sigma x_i^2 / f^2(x_i) + a_0 \Sigma x_i / f^2(x_i) = \Sigma y_i x_i / f^2(x_i)$$ First Equation

$$\partial \Sigma \varepsilon_i^2 / \partial a_0 = 2\Sigma (y_i / f(x_i) - a_1 x_i / f(x_i) - a_0 / f(x_i))(-1/f(x_i)) = 0$$

$$= \Sigma y_i / f^2(x_i) - a_1 \Sigma x_i / f^2(x_i) - a_0 \Sigma 1 / f^2(x_i) \implies$$

$$a_1 \Sigma x_i / f^2(x_i) + a_0 \Sigma 1 / f^2(x_i) = \Sigma y_i / f^2(x_i)$$ Second Equation

APPENDIX B

SOLUTION FOR BETA PARAMETERS

Given the first four additive moments G_1, G_2, G_3, G_4 the parameters of the Beta PDF (α, β, a, b) can be found as follows:

(1) $B1 = (G_3)^2/(G_2)^3$, $B2 = (G_4/(G_2)^2) + 3$

(2) $R = 6*(B2 - B1 - 1)/(6 + 3*B1 - -2*B2)$

(3) $M1 = 0.5*((R - 2) + R*(R + 2)*(B1/(B1*(R + 2)^2 + 16*(R + 1)))^{0.5}$

(4) $M2 = 0.5*((R - 2) - R*(R + 2)*(B1/(B1*(R + 2)^2 + 16*(R + 1)))^{0.5}$

(5) If $G_3 < 0$, then $\alpha =$ the larger of M1 or M2.

If $G_3 > 0$, then $\alpha =$ the smaller of M1 or M2

β then equals the other value

(6) $b = 0.5*(G_2*(B1*(R + 2)^2 + 16*(R + 1)))^{0.5}$

(7) $a = G_1 - \alpha/\beta *b/(1 + \alpha/\beta)$

APPENDIX C

INPUT AND OUTPUT FOR TRADEOFF STUDY

C.1 Input for CRPP Ship

SHIP DESCRIPTION: SHIP A: TTFF WITH CRP: NO VARIANCE IN PAYLOAD

HULL FORM GEOMETRY				
LBP	437			
LBP/B	8.5			
LBP/D	11.18			
T/D	0.493			
MAX SEC COEF	0.803			
HULL VOLUME	598710			
DKHS VOLUME	113744	8E+06	0	-4E+13
STK HEIGHT	20			
DKHS MTRL TYPE IND	STEEL			
DKHS STRUCT DENSITY	4.18	0.0087	0.0003	-0.000
SHIP REQ				
ENDURANCE	4500			
ENDURANCE SPEED	20			
PROPULSION PLANT				
MAIN ENGINE				
MAIN ENG SIZE IND	1	0=CALC 1=GIVEN		
MAIN NO ENG	2			
MAIN ENG TYPE IND	2	2=GT		
MAIN CONT PWR AVAIL	26250			
MAIN CONT RPM	3600			
MAIN ENG SPEC WT	1.99	0	0	0
MAIN CONT PWR REQ	21000			
SEC ENGINE				
SEC ENG SIZE IND	0	0=CALC 1=GIVEN		
SEC NO ENG	0			
SEC ENG TYPE IND	0	2=GT		
SEC CONT PWR AVAIL	0			
SEC CONT RPM	0			
SEC ENG SPEC WT	0	0	0	0
TRANSMISSION				
TRANS TYPE IND	1	1=MECH		
GEAR K FAC	185	28.57	71.43	-306.1
MACHINERY ROOM				
MACHY BOX VOL IND	0	0=CALC 1=GIVEN		
MACHY BOX VOL ARRAY (2X1)	125600			
	0			
MAIN ENG CG IND	0	0=CALC 1=GIVEN		
MAIN ENG CG ARRAY (2X1)	0.57			
	0.56			
SEC ENG CG IND	0	0=CALC 1=GIVEN		
SEC ENG CG ARRAY (2X1)	0			
	0			
POWERING				
NO PROP SHAFTS	2			
DESIGN DHP	20475			

```
    ENDURANCE DHP                     4646.8
  PROPELLER
    PROP TYPE IND                          2 1=FP 2=CPP
    PROP DIA                           16.35
    DESIGN PROP RPM                      140
    PROP LOC ARRAY (2X1)              0.9496
                                      0.0502
ELECTRIC PLANT
    GEN KW                              2000
    NO SS GEN                              4
    SS ENG TYPE IND                        1 1=GT 2=DIESEL
    AVG 24 HR ELECT LOAD                2801
    FREQ CONV IND                          0 0=NEW 1=OLD
COMMAND+SURVEILLANCE
  SONAR SYSTEM
    SONAR DOME IND                         1 0=NONE 1=PRESENT
    SONAR WT ARRAY (4X1)                   0       0       0       0
                                         210       0       0       0
                                         200       0       0       0
                                           0       0       0       0

AUXILIARY SYSTEMS
    VENT SYS IND                         STD
    FAN COIL IND                     PRESENT
    COLL PROTECT SYS IND                FULL
    NO AUX BOILERS                      NONE
    FIREMAIN SYS IND                     NEW
    PRAIRE MASK SYS IND                 NONE
    ROLL FIN AREA                         70
    NO FIN PAIRS                           1
    UNREP GEAR IND                    STREAM
    NO ANCHORS                             2
    POLLUTION CNTL IND               PRESENT


OUTFIT+FURNISHINGS
    UNIT COMMANDER IND                  NONE
    CREW ACCOM ARRAY (3X1) OFF            29
                           CPO            21
                           ENL           251
    HAB OUTFIT IND                    MODERN
    STOWAGE TYPE IND                  VIDMAR

WEIGHT MARGINS
    GROWTH WT MARGIN                       0


FULL LOADS
  STORES
    STORES PERIOD ARRAY (4X1)             45
```

```
                                   30
                                   45
                                   45
  FUELS+LUBRICANTS
    USABLE FUEL WT               1080.3
    BALLAST FUEL FRAC                 0

BASELINE GROUP 100 WEIGHTS
                              SWBS
                              ====
                               110   454.0
                               120   145.5
                               130   362.7
                               160    33.9
                               170    11.6
                               190    14.1
```

C.2 Output for CRPP Ship

```
                   CALCS FOR BETA PARAMETERS
                   =========================

FULL LOAD          5654.854 583.6618 41.43845 -42435.5
               B1: 0.000008            ALPHA: 21.46559
               B2: 2.875431             BETA: 21.69590
                R: 45.16149              LOW: 5491.585
               M1: 21.69590            RANGE: 328.2889
               M2: 21.46559

LIGHT SHIP         4150.800 583.6479 41.43845 -42435.5
               B1: 0.000008            ALPHA: 21.46444
               B2: 2.875425             BETA: 21.69474
                R: 45.15919              LOW: 3987.538
               M1: 21.69474            RANGE: 328.2768
               M2: 21.46444

WT GROUP 100       1495.511 458.0834 42.62560 -41615.1
               B1: 0.000018            ALPHA: 12.54361
               B2: 2.801681             BETA: 12.70661
                R: 27.25023              LOW: 1382.485
               M1: 12.70661            RANGE: 227.5207
               M2: 12.54361

WT GROUP 200       483.5223 65.17457 -1.18714 -693.965
               B1: 0.000005            ALPHA: 15.91967
               B2: 2.836626             BETA: 15.80435
                R: 33.72402              LOW: 435.7768
               M1: 15.91967            RANGE: 95.14516
               M2: 15.80435
```

```
WT GROUP 300    313.4707 21.21031          0 -83.6595
           B1:         0            ALPHA: 13.63244
           B2: 2.814039              BETA: 13.63244
            R: 29.26489               LOW: 288.1344
           M1: 13.63244             RANGE: 50.67259
           M2: 13.63244

WT GROUP 400    652.3619 1.662644          0 -0.69824
           B1:         0            ALPHA: 9.377075
           B2: 2.747412              BETA: 9.377075
            R: 20.75415               LOW: 646.3478
           M1: 9.377075             RANGE: 12.02820
           M2: 9.377075

WT GROUP 500    648.6317 24.79406          0 -29.1907
           B1:         0            ALPHA: 60.67878
           B2: 2.952515              BETA: 60.67878
            R: 123.3575               LOW: 593.1040
           M1: 60.67878             RANGE: 111.0554
           M2: 60.67878

WT GROUP 600    427.2609 12.72076          0 -12.8445
           B1:         0            ALPHA: 35.29460
           B2: 2.920623              BETA: 35.29460
            R: 72.58921               LOW: 396.6650
           M1: 35.29460             RANGE: 61.19186
           M2: 35.29460

WT GROUP 700    130.0412 0.002103          0 -0.00000
           B1:         0            ALPHA: 8.449543
           B2: 2.726015              BETA: 8.449543
            R: 18.89908               LOW: 129.8366
           M1: 8.449543             RANGE: 0.409197
           M2: 8.449543

LOADS           1504.053 0.013925          0 -0.00007
           B1:         0            ALPHA: 5.756395
           B2: 2.636645              BETA: 5.756395
            R: 13.51279               LOW: 1503.603
           M1: 5.756395             RANGE: 0.899113
           M2: 5.756395
```

C.3 Input for RRG Ship

SHIP DESCRIPTION:	SHIP B: TTFF WITH RRG: NO VARIANCE IN PAYLOAD				
HULL FORM GEOMETRY					
LBP	435				
LBP/B	8.5				
LBP/D	11.18				
T/D	0.493				
MAX SEC COEF	0.803				
HULL VOLUME	590528				
DKHS VOLUME	112685	2E+07	-3E+10	-1E+14	
STK HEIGHT	20				
DKHS MTRL TYPE IND	STEEL				
DKHS STRUCT DENSITY	4.18	0.0087	0.0003	-0.000	
SHIP REQ					
ENDURANCE	4500				
ENDURANCE SPEED	20				
PROPULSION PLANT					
MAIN ENGINE					
MAIN ENG SIZE IND	1	0=CALC	1=GIVEN		
MAIN NO ENG	2				
MAIN ENG TYPE IND	2	2=GT			
MAIN CONT PWR AVAIL	26250				
MAIN CONT RPM	3600				
MAIN ENG SPEC WT	1.99	0	0	0	
MAIN CONT PWR REQ	21000				
SEC ENGINE					
SEC ENG SIZE IND	0	0=CALC	1=GIVEN		
SEC NO ENG	0				
SEC ENG TYPE IND	0	2=GT			
SEC CONT PWR AVAIL	0				
SEC CONT RPM	0				
SEC ENG SPEC WT	0	0	0	0	
TRANSMISSION					
TRANS TYPE IND	1	1=MECH			
GEAR K FAC	185	28.57	71.43	-306.1	
MACHINERY ROOM					
MACHY BOX VOL IND	0	0=CALC	1=GIVEN		
MACHY BOX VOL ARRAY (2X1)	125600				
	0				
MAIN ENG CG IND	0	0=CALC	1=GIVEN		
MAIN ENG CG ARRAY (2X1)	0.57				
	0.56				
SEC ENG CG IND	0	0=CALC	1=GIVEN		
SEC ENG CG ARRAY (2X1)	0				
	0				
POWERING					
NO PROP SHAFTS	2				
DESIGN DHP	20580.				
ENDURANCE DHP	4245.3				

```
  PROPELLER
    PROP TYPE IND                        1  1=FP  2=CPP
    PROP DIA                         16.34
    DESIGN PROP RPM                    140
    PROP LOC ARRAY (2X1)            0.9496
                                    0.0502
ELECTRIC PLANT
    GEN KW                            2000
    NO SS GEN                            4
    SS ENG TYPE IND                      1  1=GT  2=DIESEL
    AVG 24 HR ELECT LOAD              2789
    FREQ CONV IND                        0  0=NEW  1=OLD
COMMAND+SURVEILLANCE
  SONAR SYSTEM
    SONAR DOME IND                       1  0=NONE  1=PRESENT
    SONAR WT ARRAY (4X1)                 0        0        0        0
                                       210        0        0        0
                                       200        0        0        0
                                         0        0        0        0

AUXILIARY SYSTEMS
    VENT SYS IND                       STD
    FAN COIL IND                   PRESENT
    COLL PROTECT SYS IND              FULL
    NO AUX BOILERS                    NONE
    FIREMAIN SYS IND                   NEW
    PRAIRE MASK SYS IND               NONE
    ROLL FIN AREA                       70
    NO FIN PAIRS                         1
    UNREP GEAR IND                  STREAM
    NO ANCHORS                           2
    POLLUTION CNTL IND             PRESENT


OUTFIT+FURNISHINGS
    UNIT COMMANDER IND                NONE
    CREW ACCOM ARRAY (3X1) OFF          29
                           CPO          21
                           ENL         251
    HAB OUTFIT IND                  MODERN
    STOWAGE TYPE IND                VIDMAR

WEIGHT MARGINS
    GROWTH WT MARGIN                     0



FULL LOADS
  STORES
    STORES PERIOD ARRAY (4X1)           45
                                        30
                                        45
```

```
                                             45
  FUELS+LUBRICANTS
    USABLE FUEL WT                       1037.4
    BALLAST FUEL FRAC                         0

BASELINE GROUP 100 WEIGHTS
                              SWBS
                              ====
                               110   445.5
                               120   144.2
                               130   359.0
                               160    33.4
                               170    11.5
                               190    13.9
```

C.4 Output for RRG Ship

```
               CALCS FOR BETA PARAMETERS
               =========================

FULL LOAD      5545.922 597.4037 -160.765 -40636.5
           B1: 0.000121            ALPHA: 24.30157
           B2: 2.886137             BETA: 23.31105
            R: 49.61263              LOW: 5368.384
           M1: 24.30157            RANGE: 347.8403
           M2: 23.31105

LIGHT SHIP     4087.054 597.3899 -160.765 -40636.5
           B1: 0.000121            ALPHA: 24.30034
           B2: 2.886132             BETA: 23.30986
            R: 49.61020              LOW: 3909.522
           M1: 24.30034            RANGE: 347.8280
           M2: 23.30986

WT GROUP 100   1473.182 473.4162 -155.686 -39769.2
           B1: 0.000228            ALPHA: 14.71272
           B2: 2.822556             BETA: 14.03756
            R: 30.75028              LOW: 1347.671
           M1: 14.71272            RANGE: 245.2619
           M2: 14.03756

WT GROUP 200   453.2787 64.03193 -4.96252 -743.109
           B1: 0.000093            ALPHA: 14.24950
           B2: 2.818757             BETA: 13.83044
            R: 30.07994              LOW: 407.9979
           M1: 14.24950            RANGE: 89.22993
           M2: 13.83044

WT GROUP 300   312.0422 21.17580 -0.05039 -83.0806
           B1: 0.000000            ALPHA: 13.70281
```

```
                B2: 2.814723                  BETA: 13.68118
                 R: 29.38399                   LOW: 286.6567
                M1: 13.70281                 RANGE: 50.73089
                M2: 13.68118

WT GROUP 400        651.6513 1.626482 -0.00004 -0.66803
                B1: 0.000000                 ALPHA: 9.380425
                B2: 2.747477                  BETA: 9.379832
                 R: 20.76025                   LOW: 645.7019
                M1: 9.380425                 RANGE: 11.89834
                M2: 9.379832

WT GROUP 500        642.7324 24.49025 -0.04687 -27.9067
                B1: 0.000000                 ALPHA: 62.04485
                B2: 2.953471                  BETA: 61.90655
                 R: 125.9514                   LOW: 586.9111
                M1: 62.04485                 RANGE: 111.5181
                M2: 61.90655

WT GROUP 600        424.1345 12.64710 -0.01897 -12.6033
                B1: 0.000000                 ALPHA: 35.60658
                B2: 2.921204                  BETA: 35.53926
                 R: 73.14585                   LOW: 393.4832
                M1: 35.60658                 RANGE: 61.24479
                M2: 35.53926

WT GROUP 700        130.0328 0.002092 -0.00000 -0.00000
                B1: 0.000000                 ALPHA: 8.508138
                B2: 2.727410                  BETA: 8.502982
                 R: 19.01112                   LOW: 129.8281
                M1: 8.508138                 RANGE: 0.409290
                M2: 8.502982

LOADS               1458.867 0.013805 -0.00000 -0.00006
                B1: 0.000000                 ALPHA: 5.890599
                B2: 2.642305                  BETA: 5.883463
                 R: 13.77406                   LOW: 1458.415
                M1: 5.890599                 RANGE: 0.903244
                M2: 5.883463
```

C.5 Payload

PAYLOAD DESCRIPTION

	PAYLOAD NAME	SWBS	MEAN	VARIANCE	SKEW	KURTOSIS
1	COMMAND AND CONTROL	W410	9.70	0	0	0
2	EXTERIOR COMMS	W440	14.30	0	0	0
3	SURFACE SEARCH AND IFF	W450	4.80	0	0	0
4	NAVIGATION RADAR	W450	0.10	0	0	0
5	IR DETECTOR	W450	1.00	0	0	0
6	TOWED ARRAY	W460	50.00	0	0	0
7	ASW ELECTRONICS	W460	90.00	0	0	0
8	ACTIVE ECM	W470	3.50	0	0	0
9	ACOUSTIC DECOY	W470	2.30	0	0	0
10	MK-92 FCS	W480	5.00	0	0	0
11	76MM GUN	W710	34.90	0	0	0
12	TWO CIWS	W710	11.00	0	0	0
13	32 CELL VLS	W720	64.50	0	0	0
14	16 CELL VL SEASPARROW	W720	11.50	0	0	0
15	SRBOC	W720	2.20	0	0	0
16	MK-32 SVTT	W750	4.00	0	0	0
17	76MM AMMO	WF21	6.60	0	0	0
18	12000 RDS 20MM AMMO	WF21	9.20	0	0	0
19	32 ASROC/HARPOON	WF21	55.00	0	0	0
20	16 SEASPARROW	WF21	3.90	0	0	0
21	2 RSL SRBOC	WF21	2.40	0	0	0
22	TORPEDOES IN TUBES	WF21	1.40	0	0	0
23	THREE LAMPS III	WF23	26.70	0	0	0
24	LAMPS HANDLING AND STOWAGE	W588	15.00	0	0	0
25	LAMPS SUPPORT	WF26	12.00	0	0	0
26	LAMPS JP-5	WF42	95.00	0	0	0
27	LAMPS TORPEDOES	WF22	12.00	0	0	0
28	SONOBUOYS	WF26	12.00	0	0	0

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Thesis
W22291 Walsh
c.1
An improved method of risk analysis for the naval ship design process.